Mon. Not. R. astr. Soc. (1968) **140,** 71–79.

EXCITATION TEMPERATURES OF THE WOLF–RAYET STARS

M. K. V. Bappu and K. S. Ganesh

(Received 1967 December 5)

Summary

Hydrogenic transitions of C IV are used to derive excitation temperatures for Wolf-Rayet stars of the WC sequence. The line intensities of such transitions in HD 165763, HD 192641, HD 192103 and HD 184738 are derived from measures of equivalent widths on spectra of 10 Å/mm and 20 Å/mm dispersion. These intensities are corrected for continuum values derived by the method of photoelectric spectrophotometry. This technique of temperature determination yields values of temperature unaffected by non-LTE effects in the Wolf-Rayet atmosphere. The mean values obtained are as follows: HD 165763 (WC6) 56 900° ± 2600°; HD 192641 (WC7) 51 600° ± 6300°; HD 192103 (WC7) 37 300° ± 3400°; HD 184738 (WC8) 24 000° ± 4100°K.

The intensities of the (4*n*) series of He II are used to derive an excitation temperature of the WN7 star HD 192163, using intensities measured on a single high dispersion Mount Wilson coudé spectrogram. The value derived is 32 400° ± 4200°. A Balmer contribution to the emission of the Pickering series is obvious, though the hydrogen emission present is little.

Arguments are presented to show that the (5–9) transition 8236 Å and the (5–15) transition 6406 Å can be utilized on high dispersion spectra for obtaining reliable excitation temperatures of stars of the WN sequence.

1. *Introduction.* Numerous investigators have attempted over the years, to derive the temperatures of the Wolf-Rayet stars. Gerasimovic (1933) made the first systematic attempt to derive effective temperatures from continuum intensities. Petrie (1947) in a later study obtained a value of effective temperature of the order of 18 000°K. Several uncertainties existed in these measurements that originated not only from the techniques of photographic photometry but also from the corrections necessitated by interstellar reddening. Miss Underhill (1959) has shown that the continuum of Wolf-Rayet spectra in the neighbourhood of 3500 Å, simulates the intensity gradient as seen in the late O or early B type stars. She argued that it is most likely that the effective temperatures of the Wolf-Rayet stars fall in the neighbourhood of 25 000° to 35 000°. Kuhi (1966) has recently carried through photo-electric scans of the energy distribution in several Wolf-Rayet stars in the northern hemisphere. Avoiding the locations of strong emission lines, Kuhi detected the rather peculiar behaviour that the colour temperature decreases with increasing wavelength. The values of colour temperature of the WN and WC stars resemble those of the O and early B stars in the blue and ultra-violet regions but become considerably lower as one proceeds to the red and near infra-red regions.

The emission spectra have been utilized for estimating the ionization and excitation temperatures. Making use of the Zanstra–Menzel mechanism of photo-ionization and subsequent recombination, Beals (1934) obtained a range in ionization temperatures for the Wolf-Rayet stars from 60 000° to 110 000°. An extension of these studies by Vorontsov-Velyaminov (1946) modified these values down to the range 25 000° to 84 000°. These temperatures depend on the ion used for the determination and this led Beals to postulate the hypothesis that the Wolf-Rayet atmosphere is stratified, with the higher ionization stages originating closer to the surface of the star. Aller (1943) derived excitation temperatures on the assumption of thermodynamic equilibrium, by comparing intensities of pairs of lines of different ionization levels. These excitation temperatures ranged from 20 000° to 120 000°. Weenen (1950) in a rediscussion of Aller's data found that the excitation temperatures have little or no significance. Bappu (1958) employed the hydrogenic transitions of C IV to derive excitation temperatures of WC7 and WC8 stars. The values for these two spectral types were 49 000° and 27 000° respectively. The argument employed by Bappu has been that the C IV hydrogenic transitions originate from the higher levels of the atom and as such abnormal populations of the various levels, caused by deviations from thermodynamic equilibrium, would be non-existent.

2. *The problem of temperature determination.* The original Beals postulate of the Zanstra–Menzel mechanism of photo-ionization with subsequent recombination is an over simplified case of the actual situation. The Wolf-Rayet stars are high temperature objects and necessarily may have an intense UV spectrum. However, the first rocket evaluations of the UV region of Gamma–Velorum show that while there seems an increased intensity at 1600 Å compared to that at 2000 Å, the flux measured, falls short of the flux expected of a model atmosphere of $T_e = 28\,470$ and $\log g = 3{\cdot}8$. Hence the Wolf-Rayet star may not necessarily be as rich in shortwave radiation as we had expected it to be earlier.

In view of the extreme predominance of mass motions, it is also obvious that collision effects will play a significant role. We are still not clear about the nature of the atmospheric support. If as Thomas (1949) pointed out, microscopic motion alone exists in the form of a $T_e > T_r$, where T_e and T_r are the kinetic and radiation temperatures respectively, then a positive excitation gradient follows as a direct consequence, similar to the solar chromosphere and corona. We have increasing evidence to show that the higher excitation lines originate farther from the star's 'photosphere'. As such the picture of pure ionization and subsequent recombination alone, by virtue of radiation from an ultra-hot core may not be very realistic.

The derivation of temperatures representative of the populations of the excited levels, has the characteristic of being non-commital regarding the nature of the excitation. Aller (1943) pioneered this approach for the Wolf-Rayet stars. But his selection of emission lines was confined to every single features for which line strengths could be computed and which could have an intensity measure from low dispersion slitless spectra. It is increasingly obvious that the profile of a Wolf-Rayet emission line is inevitably a summation of blends, and that great care is necessary to select the lines for study in any attempt to infer temperatures from line ratios. In fact, judging from the high dispersion spectra that we have studied, we consider that there are very few lines that can come under such a classification of being free of blends.

A second difficulty that is apparent in Aller's study is that his results have been affected by the absence of thermodynamic equilibrium in the atmosphere. Thomas has shown (1949b) that Aller's variation of $N_i N_e$ with ionization may not indicate stratification in the Wolf-Rayet atmosphere but could also be due to departures from thermodynamic equilibrium. Hence in choosing any technique of excitation temperature determination in the Wolf-Rayet atmosphere, one needs to be careful to avoid abnormal population effects of the levels caused by non-LTE effects.

Such an approach was first employed by Bappu (1958) with high dispersion spectra. This work was greatly facilitated by the identification by Edlen (1956) of the hydrogenic transitions of C IV. The use of these C IV transitions in a determination of the state of excitation of the C IV ions has two attractive aspects. Firstly one can employ the Einstein $A_{n'n}$ coefficients calculated already for hydrogen. Secondly the C IV transitions identified in the Wolf-Rayet stars originate from transitions between the higher levels. Bappu used the (7*n*) series, more specially the transitions 4229 Å (7–12), 3567 Å (7–15) and 3451 Å (7–16). In discussions of deviations from thermodynamic equilibrium one considers the ratio b_n between the actual population of the *n*th level of an ion and the population under conditions of thermodynamic equilibrium. In studies of the excitation temperatures, these b_n parameters necessarily play a role and if we are to derive any values that represent a situation close to reality, we have to account for them satisfactorily. This is the main feature of Bappu's analysis that was confined to just two stars. We follow a similar line of reasoning and extend these results in the present study.

3. *The method of temperature determination.* Let us consider two emission lines that are the result of transitions $n' \rightarrow n$ and $n'' \rightarrow n$. We then have

$$\frac{I_{n'n}}{I_{n''n}} = \frac{N_{n'}A_{n'n}h\nu_{n'n}}{N_{n''}A_{n''n}h\nu_{n''n}}$$

where $N_{n'}$ and $N_{n''}$ are the populations of levels n' and n'' respectively and $A_{n'n}$ the Einstein coefficient for spontaneous transition $n' \rightarrow n$.

Since,

$$N_n = b_n N_i N_e \frac{h^3}{(2\pi m k T_e)^{3/2}} \frac{\bar{\omega}_n}{\bar{\omega}_i} \exp\left(\frac{\chi_n - \chi_i}{kT}\right)$$

we have

$$T = \frac{(\nu_{n'} - \nu_{n''})\dfrac{h}{k}}{\ln\left(\dfrac{I_{n'n}}{I_{n''n}}\right) - \ln\left(\dfrac{A_{n'n}\nu_{n'}\bar{\omega}_{n'}}{A_{n''n}\nu_{n''}\bar{\omega}_{n''}}\right) - \ln\left(\dfrac{b_{n'}}{b_{n''}}\right)}.$$

We identify T, the excitation temperature, with the kinetic temperature T_e in the limiting case. The temperature determinations call for a knowledge of the b_n ratios. However, the advantage of using the hydrogenic transitions of C IV is that the b_n ratio can be taken as 1. Thomas (1949a) and Chamberlain (1953) have shown that for large values of n, $b_n \rightarrow 1$. Also, b_n for any n, decreases rapidly with large nuclear charge. Hence, if we consider any transitions involving n greater than 5 and for an ion with larger nuclear charge than hydrogen, the ratio $b_{n'}/b_{n''}$ is obviously 1. We can then derive safely by simple astrophysical formulae the temperature parameter, just by a judicious choice of lines.

One measures the intensity of the emission line conventionally in terms of the intensity of the continuum. This has a physical significance in measures of the equivalent widths of absorption lines. However, in the case of emission lines one can compare the differences between the energies radiated in two emission lines only after one can normalize the two continua against which the emission intensity is measured. Bappu's analysis of the C IV hydrogenic transitions needs refinement in this specific aspect. He could not make these corrections with certainty because, accurate spectrophotometry of the continuum of the Wolf-Rayet stars did not exist in the literature. Recently, a significant contribution to this problem has been the painstaking work of Kuhi (1966) who measured for a few bright Wolf-Rayet stars, the intensities photo-electrically at selected locations in the spectra that were free of emission lines. A factor that has to be compensated for, is the correction to interstellar reddening. Kuhi has done this on the basis of our knowledge of this feature today.

4. *The temperatures of stars of the WC sequence.* In Table I, we give the measured equivalent widths of those C IV hydrogenic transitions in the Wolf-Rayet carbon sequence stars that are relatively free of blends. Kuhi has given unreddened fluxes for HD 165763, HD 192641 and HD 192103. The intensities I of the emission lines are the rectified ones after applying the Kuhi correction factors for each star. These are also given for each star and represent interpolated values from a plot of the continuum fluxes measured by him. Kuhi has no determination of the energy distribution in HD 184738, the nucleus of Campbell's hydrogen envelope star, presumably because of its faintness. We have therefore, used correction factors appropriate to black body radiation at 15 000°K. The basis of this choice is to some extent arbitrary. Firstly, the colour temperatures of Wolf-Rayet stars, as determined in the 4000–5000 Å region are much lower than their excitation temperatures, almost by a factor two. Bappu's earlier estimate of temperature for this star was 28 000°. Also the stars HD 192103 and HD 192641 have estimates of the colour temperatures by Kuhi of the order of 20 000° in the 5000 Å region. Hence our estimate of 15 000° for HD 184738 is likely to represent an upper limit of colour temperature.

The atomic parameters utilized are given below:

λ	n–n'	$A_{nn'}$ for hydrogen	ν
5470	7–10	$38 \cdot 8 \times 10^3$	$0 \cdot 5481 \times 10^{15}$
4229	7–12	$12 \cdot 4 \times 10^3$	$0 \cdot 7089 \times 10^{15}$
3689	6– 9	$70 \cdot 3 \times 10^3$	$0 \cdot 8125 \times 10^{15}$
3567	7–15	$3 \cdot 512 \times 10^3$	$0 \cdot 8405 \times 10^{15}$
3450	7–16	$2 \cdot 463 \times 10^3$	$0 \cdot 8690 \times 10^{15}$

Table II gives the temperatures obtained for the different lines ratios. The mean values together with the probable errors are also given. A few features need discussion. In HD 192103 the values of temperature given by line ratios involving 3687 Å are conspicuously low. This is because the intensity of 3687 Å is very low. The value measured from the spectrum is low by a factor of 1·4; the value of intensity of 3687 Å needs to be 36·2 instead of 26·5, which is the measured value. An error of this magnitude cannot be made in the photometry. Clearly the cause is elsewhere and the immediate suspicion is of weak Balmer line absorption that suppresses the true intensity. The profile of this line in HD 192103 is distinctly different from that seen in the other carbon stars, thus strengthening the argument.

TABLE I

The emission line intensities in the WC sequence

	Transition	HD 165763 WC 6			HD 192641 WC 7			HD 192103 WC 7			HD 184738 WC 8		
		$W(A)$	Kuhi factor	I	$W(A)$	Kuhi factor	I	$W(A)$	Kuhi factor	I	$W(A)$	Continuum correction 15 000°	I
5470	7–10	33·78	1·0	33·78	10·57	1·0	10·57	19·5	1·0	19·5	—	1·0	—
4229	7–12	7·00	1·368	9·58	2·49	1·368	3·409	9·72	1·459	14·18	3·84	1·192	4·58
3689	6– 9	18·35	1·528	28·00	1·204	1·614	10·013	16·43	1·614	26·52	—	—	—
3567	7–15	3·52	1·614	5·68	1·32	1·592	2·10	5·6	1·542	8·64	2·61	1·253	3·27
3450	7–16	—	—	—	—	—	—	4·57	1·644	7·51	2·25	1·256	2·83

TABLE II

Temperatures of the WC stars

Line ratio	Temperature (°K) HD 165763	 HD 192641	 HD 192103	 HD 183738
$\frac{4229}{5470}$	—	—	38298*	—
$\frac{3687}{5470}$	—	—	14280	—
$\frac{3567}{5470}$	—	—	40157*	—
$\frac{3450}{5470}$	—	—	38354*	—
$\frac{3687}{4229}$	61154	62497	20391	—
$\frac{3567}{4229}$	51847	39518	42583*	20543
$\frac{3450}{4229}$	—	—	38104*	21683
$\frac{3567}{3687}$	57733	52664	24363	—
$\frac{3450}{3687}$	—	—	24355	—
$\frac{3450}{3567}$	—	—	26409*	29675
Mean	56911 ±2600	51560 ±6300	37318 ±3400	23967 ±4100

* The mean temperature derived for HD 192103.

3687 Å has a line intensity that yields satisfactory temperatures in the other two stars in which it can be measured.

The intensity of 5470 Å also calls for comment. This line has been measured on the spectra of HD 165763, HD 192641 and HD 192103. Only in the case of HD 192103 does it give physically significant temperatures. In both the other stars its intensity is larger than need be to give a temperature in agreement with the other line ratios. We believe that this may be due to blending with OV 5474. This problem would exist only in the high temperature stars and hence we find it absent in HD 192103.

Our study of the four stars shows a temperature variation with spectral class as is to be expected. The temperature of HD 184738 may be raised by a thousand degrees or two when its continuum fluxes are better determined.

5. *The temperatures of stars of the WN sequence.* We are not as fortunately placed in the case of the nitrogen sequence Wolf-Rayet stars as we have been for the WC sequence. The use of the C IV hydrogenic transitions eliminated the need of new determinations of transition probabilities. It also eliminated the

consideration of problems arising from non-LTE effects, because the transitions used originate from the higher levels. An approach of this type could be used for the WN stars by employing hydrogenic transitions of NV. Edlen (1956) has listed the ones that may be expected to be present in the region of the spectrum accessible to ground-based telescopes. However, these are too weak to be identified with certainty and also fall in regions where they are likely to be heavily blended with the lines of the other ions.

We, therefore, have to depend on the helium series of emission lines for any information on these stars with the reasoning that we have applied earlier. Two series are available; the (4*n*) and (5*n*) series are seen on the spectra that one usually gets in the range 4000–9000 Å. The difficulties that we are likely to face are two-fold. Firstly, there can be possible Balmer line contribution to the alternate members of the Pickering series. Secondly, we have no means of determining the b_n ratio.

In the following discussion we use intensity data, only of HD 192163, since, it has the strongest lines among the stars for which we have high dispersion spectra. This would not only improve photometric accuracy but also minimize blending possibilities by virtue of the star's intrinsically narrower line widths. Intensity measures made of the 4*n* series have been confined to 6560 Å (4–6), 5411 Å (4–7), 4860 Å (4–8), 4542 Å (4–9), 4340 Å (4–10) and 4200 Å (4–11). These are listed below along with the Kuhi continuum corrections.

λ	Transition	*W*(*A*)	Kuhi factor	*I*
6560	4– 6	161·3	0·624	100·6
5411	4– 7	75·0	0·822	61·6
4860	4– 8	46·9	1·000	46·9
4542	4– 9	42·0	1·086	34·7
4340	4–10	23·4	1·148	26·9
4200	4–11	17·2	1·225	21·1

On the assumption that the b_n ratios of the transitions are unity we get for the different line ratios the following values of temperature.

HD 192163

Line ratio	Temperature (°K)	Line ratio	Temperature (°K)
$\frac{4860}{5411}$	27200	$\frac{4340}{4860}$	35397
$\frac{4542}{5411}$	33036	$\frac{4200}{4860}$	33335
$\frac{4340}{5411}$	31137	$\frac{4340}{4542}$	25958
$\frac{4200}{5411}$	30639	$\frac{4200}{4542}$	26805
$\frac{4542}{4860}$	48067		

Mean = 32 397° ± 4200 °K.

1968MNRAS.140...71B

A temperature cannot be obtained from the $\frac{5411}{6560}$ ratio. Values of temperature obtained from the $\frac{4861}{6560}$, $\frac{4340}{6560}$, $\frac{4200}{6560}$ ratios are 161 000°, 75 600° and 66 000° on the assumption of a b_n ratio of unity. If we assume on the basis of the value obtained from say $\frac{4542}{5411}$ or $\frac{4200}{5411}$ that the temperature is 33 000°, we can reverse the argument and get at the b_n ratios. These turn out to be the following:

6560/5411 b_6/b_7 1·226	6560/4860 b_6/b_8 1·203	6560/4542 b_6/b_9 1·227	6560/4340 b_6/b_{10} 1·212
	5411/4860 b_7/b_8 0·9804	5411/4542 b_7/b_9 1·0000	5411/4340 b_7/b_{10} 0·9881
		4860/4542 b_8/b_9 1·020	4860/4340 b_8/b_{10} 1·007
			4542/4340 b_9/b_{10} 0·9872

The b_n ratios obtained for 6563 Å are all larger than unity. If the b_n ratio were unity and the temperature is 33 000° then the intensity of the 6560 Å emission line needs to be 82·7 instead of 100·6. An error of 18 per cent cannot be admitted in the photometry, even if it is by a photographic technique. Hence we either accept the deviation from unity of the b_n ratio or explain the enhanced intensity of 6560 Å by other means.

If the b_n ratio is not unity then we should have b_6/b_n ($n>7$) increase as n takes on larger values. Instead, we see that the b_6/b_n ratio is almost constant, with b_6/b_7 and b_6/b_9 having larger values than b_6/b_8 and b_6/b_{10}. This indicates a contamination in the intensity affecting 6560 Å, 4860 Å and 4340 Å. The conclusion seems to be that we have here a case of Balmer contribution affecting the intensities of these lines. In other words the intensities of 6560 Å, 4860 Å, 4340 Å are higher than would be expected if they were purely of the helium Pickering series. The b_n ratios b_7/b_8, b_7/b_9, b_7/b_{10} indicate that this reasoning has validity since b_7/b_8 and b_7/b_{10} are less than unity while b_7/b_9 is unity. Clearly the hydrogen contribution is obvious but the amount of hydrogen emission present is little.

We have seen that in HD 192163 the b_7/b_9 ratio derived from the ($4n$) series is unity. Hence if we can use the ($5n$) series, we should be more justified in assuming the b_n ratio to be unity. The lines identified with certainty of this series are 6310 Å (5–16), 6406 Å (5–15), 6527 Å (5–14), 6682 Å (5–13), 6891 Å (5–12), 7177 Å (5–11) and 8236 Å (5–9). Many of these lines are blended, though it is possible that 8236 Å, 7177 Å and 6406 Å can be extricated free of the blends. The temperature evaluation needs photometry on higher dispersion spectra than what we had available in the infrared. We have, therefore, computed the line ratios necessary for different temperatures, in terms of the intensity of 6406 Å, and for a b_n ratio of unity. These are in Table III. It is obvious that to determine a tem-

TABLE III

Intensity ratio $I_{5n}/I_{6406\,\text{Å}}$ *for different temperatures*

		Temperature (°K)				
n–n′	(Å)	20 000	30 000	40 000	50 000	60 000
5–16	6310	0·8364	0·8318	0·8292	0·8279	0·8269
5–15 (unit)	6406	1·0000	1·0000	1·0000	1·0000	1·0000
5–14	6527	1·1852	1·1941	1·1981	1·2005	1·2024
5–13	6683	1·4508	1·4733	1·4836	1·4906	1·4951
5–12	6891	1·7606	1·8073	1·8312	1·8457	1·8556
5–11	7177	2·1896	2·2784	2·3245	2·3524	2·3714
5– 9	8236	3·4916	3·7936	3·9557	4·0552	4·1237

perature with an accuracy of 5000° in the range 25 000–35 000 °K, the photometry will have to be done, to better than one per cent, if one uses high dispersion material available with the spectral sensitivity of the 103aF emulsion. However, if one uses the line ratio 8236 Å/6406 Å, this comes well within the accuracy of photographic photometry. It will be well worthwhile to attempt the spectrophotometry of HD 192163 at a dispersion of 20 Å/mm. The utility of the (5*n*) series cannot be exploited today by observations shortward of 6800 Å.

The high dispersion spectra were obtained by one of us (MKVB) during the tenure of a Carnegie Fellowship at the Mount Wilson and Palomar Observatories.

Kodaikanal Observatory,
Kodaikanal,
India.
1967 *December.*

References

Aller, L. H., 1943. *Astrophys. J.*, **97**, 135.
Bappu, M. K. V., 1958. *Mém. Soc. R. Sci. Liège*, **20**, 40.
Beals, C. S., 1934. *Publs Dom. astrophys. Obs.*, **6**, 125.
Chamberlain, J. W., 1953. *Astrophys. J.*, **117**, 387.
Edlen, B., 1956. Identification of emission lines in the spectra of Wolf-Rayet stars, in *Vistas in Astronomy*, ed. by A. Beer, Vol. 2, p. 1456. Pergamon Press, London.
Gerasimovic, B. P., 1929. *Harv. Obs. Cir.*, No. 339.
Kuhi, L. V., 1966. *Astrophys. J.*, **143**, 753.
Petrie, W., 1947. *Publs Dom. astrophys. Obs.*, **7**, 383.
Thomas, R. N., 1949a. *Astrophys. J.*, **109**, 480.
Thomas, R. N., 1949b. *Astrophys. J.*, **109**, 500.
Underhill, A. B., 1959. *Publs Dom. astrophys. Obs.*, **11**, 209.
Vorontsov-Velyaminov, B. A., 1946. *Astr. Zh.*, **23**, 3.
Weenen, J., 1950. *Bull. astr. Insts Neth.*, **11**, 165.