A study of the evolution of the bulges and disks of spiral galaxies in interacting and isolated environments

A Thesis

Submitted for the Degree of

Doctor of Philosophy

in the Faculty of Science

by

Ankit Kumar

Joint Astronomy Programme
Department of Physics
Indian Institute of Science
Bangalore − 560 012, INDIA

November 2022

Declaration

I hereby declare that the work reported in this doctoral thesis titled "A study of the evolution of the bulges and disks of spiral galaxies in interacting and isolated environments" is entirely original and is the result of investigations carried out by me in the Department of Physics, Indian Institute of Science, Bangalore, under the supervision of Prof. Mousumi Das at the Indian Institute of Astrophysics, Bangalore and Dr. Nirupam Roy at the Indian Institute of Science, Bangalore.

   I further declare that this work has not formed the basis for the award of any degree, diploma, fellowship, associateship or similar title of any University or Institution.

Date: November 2, 2022   Ankit Kumar

Bangalore, India   Department of Physics

                                                             Indian Institute of Science

To

My Family

Acknowledgements

To reach where I am today, there is the contribution of each and every individual. Here, in this thesis, I am extending my gratitude to all of them.

Firstly, I am very obliged to have my supervisor, Prof. Mousumi Das. Her active encouragement, fruitful scientific discussions, constant support, and freedom to work helped me a lot to dig up deep insight into my reseach topic during this journey. Her care, like my mother, always makes me feel being with family. I want to thank her for never stopping me from pursuing my hobbies throughout my Ph.D.

I thank my nominal guide, Prof. Nirupam Roy, for his help and support in my academic and professional work at the Indian Institute of Science. I am grateful for his kind nature, friendly behaviour, keeping me up to date about academics, and his instant help whenever needed.

I would like to thank my senior, Dr Sandeep Kataria, for his active discussions and insightful suggestions related to research. I am grateful to have him beside me like a brother. I am thankful to him for encouraging me to participate actively in non-academic activities as well.

I am fortunate to work with Dr Soumavo Ghosh. His beneficial suggestions and comments helped me learn to frame research professionally and work in collaboration. I thank Prof. Victor Debattista for his constructive feedback and insightful comments during one of my research projects.

I am thankful to Prof. Annapurni Subramaniam for letting me study in her group. I really appreciate her kind nature, supportive behaviour, and giving her precious time to discuss various topics in stellar astrophysics.

My sincere thank to Prof. Stacy McGaugh for his suggestions, feedback and valuable time for discussion. I sincerely thank Prof. Volker Springel and Prof. Chien Peng for their help and advice regarding codes over email communications. I express my gratitude to my coursework guide, Prof. Firoza Sutaria, for helping me learn IRAF, fv-tool, JUDE and other tools for observational science.

I extend my gratitude to faculties from various institutions in Bangalore (in alphabetical order): Prof. Banibrata Mukhopadhyay, Prof. Biman Nath, Prof. Chanda Jog, Prof. GC Anu-

Last but not least, I would like to thank those who have helped during this beautiful journey, and I forgot to mention it here.

Preface

Galaxies are vast collections of stars, gas, dust, and invisible dark matter. They are usually found in groups and clusters, where they interact can with each other gravitationally. These interactions affect the morphology and kinematics of galaxies over the course of their evolution. In this thesis, we have studied the effect of flyby interactions and dark matter distributions on the evolution of bulges and disks of spiral galaxies using N-body simulations. All of the galactic components in our simulations are treated live. We have also studied the cosmic evolution of bulges since z=0.1 using SDSS DR7 data. Additionally, the statistical properties of bulgeless galaxies in a state-of-art cosmological simulation, IllustrisTNG, have been compared with observed bulgeless galaxies in order to test our understanding of galaxy formation. In the following paragraphs, we describe our findings.

To investigate the effect of flyby interactions on the bulges, disks, and spiral arms of Milky Way mass galaxies, we modelled disk galaxies with two types of bulges: classical bulges and boxy/peanut pseudo-bulges. We have performed N-body simulations of galaxy flybys of 10:1 and 5:1 mass ratios with varying pericenter distances. Using photometric and kinematic bulge-disk decompositions of the major galaxy at regular time steps, we found that the disks become shorter and thicker during flyby interactions. Flyby induced spiral arms are transient. They form just after pericenter passage, quickly reach maximum strength, and then slowly decay after reaching maximum strength. There is no effect of flyby interactions on the classical bulges. However, pseudo-bulges in the host galaxy become dynamically hotter at the cost of the disk. We also found no effect of flyby interactions on the strength and the formation time of bar buckling.

We found that flyby induced spiral arms are density waves in nature in contrast to material arms. These spiral arms show two winding phases: the initial rapid winding phase and the subsequent slow winding phase. We confirmed that the spiral arms are the main drivers of the observed wave-like vertical breathing motion in the Milky Way, and the effect of tidal interactions does not directly induces breathing motion. The strongest spiral arms produce the

largest breathing motion. For a given pericenter distance, the co-planar prograde orbit produces the strongest spiral arms and the largest breathing motion. On the other hand, co-planar retrograde orbits produce no spiral arms and hence no breathing motion.

In another work, we have simulated the effect of dark matter halo shape on the formation of boxy/peanut/x-shape bulges via bar buckling. We found that the presence of oblate dark matter halos delay bar formation and so bar buckling is also delayed. All the models show two buckling events but the most extreme prolate halo exhibits three distinct buckling events. As a result of multiple buckling events, the boxy/peanut structures in prolate halos show the maximum thickness. Since ongoing buckling events are rarely observed, our study suggests that the most barred galaxies may have oblate or spherical halos rather than prolate halos.

We have also studied the cosmic evolution of bulges since z=0.1 using SDSS data. We compared two well know bulge classification schemes based on the Sérsic index and Kormendy relation. We have shown that both schemes have their own drawbacks and suggest an alternative scheme which combines these two schemes. We have also found that disk-like pseudo-bulges are growing in number as the Universe is getting older. The pseudo-bulges appear optically diffuse compared to classical bulges and are commonly found in low mass galaxies. In the local volume, pseudo-bulges overcome the classical bulges even in bulge dominated galaxies, and so more than 75% of the local volume is rotation dominated.

Finally, we have tested galaxy formation models of the state-of-art cosmological simulation, Illustris TNG, using bulgeless galaxies. We selected Illustris TNG50 galaxies having mass greater than $10^9 M_{\odot}$ and performed photometric decomposition to find bulgeless galaxies. We found that the bulgeless galaxies in TNG50 are metal-poor and have high specific angular momentum as compared to the galaxies with bulges and fall at the lower end of the baryonic to dark matter mass ratio. We also found that the fraction of bulgeless galaxies in TNG50 is equivalent to that determined from observations of galaxies. Thus the TNG galaxy formation model is capable of producing observed characteristics of bulgeless galaxies in the low redshift Universe.

List of Publications

1. **Kumar, A.**, Ghosh, S., Kataria, S. K., Das, M., and Debattista, V. P., 2022, *Excitation of vertical breathing motion in disc galaxies by tidally-induced spirals in fly-by interactions*, Monthly Notices of the Royal Astronomical Society, 516, 1, 1114-1126.

2. **Kumar, A.**, and Kataria, S. K., 2022, *Growth of disc-like pseudo-bulges in SDSS DR7 since z = 0.1*, Monthly Notices of the Royal Astronomical Society, 514, 2, 2497-2512.

3. **Kumar, A.**, Das, M., and Kataria, S. K., 2022, *The effect of dark matter halo shape on bar buckling and boxy/peanut bulges*, Monthly Notices of the Royal Astronomical Society, 509, 1, 1262-1268.

4. **Kumar, A.**, Das, M., and Kataria, S. K., 2021, *Galaxy flybys: evolution of the bulge, disc, and spiral arms*, Monthly Notices of the Royal Astronomical Society, 506, 1, 98-114.

5. **Kumar, A.**, Das, M., and Kataria, S. K., 2020, *The evolution of bulges of galaxies in minor fly-by interactions*, Proceedings of the International Astronomical Union, 353, 166-167.

6. **Kumar, A.**, and Das, M., 2022, *Bulgeless galaxies in the Illustris TNG50 simulations: A test for angular momentum problem*, (In preparation).

Table of contents

List of figures

List of tables

"In all chaos there is a cosmos, in all disorder a secret order."

– **Carl Gustav Jung**

1

Introduction

Not all the point-like sources we observe in the night sky are stars. Several of them are actually groups of stars known as stellar systems. Their distances from us are too large to identify without zooming into the vastness of the Universe with high resolution telescopes. Galaxies are the stellar systems comprising a huge number of stars ranging from 10^5 to 10^{12}. Apart from a copious number of stars, galaxies also consist of gas, dust, and enormous amount of dark matter. The self-gravitational force of the galaxy holds the stars together. Galaxies can be considered as the fundamental constituents of the Universe because they are the main visible sources that contribute to the observable Universe (Gott et al., 2005). There are trillions of galaxies in our observable Universe (Conselice et al., 2016). The hazy patch of light seen stretching from north to south in the night sky is our home galaxy, the Milky Way or simply the Galaxy (with capital 'G').

1.1   A brief history of the discovery of our Galaxy

Until 1925, it was not clear whether the Milky Way was the only galaxy or was instead one of several million such galaxies in the Universe. This question gave rise to a debate in astronomy known as "The Great Debate" (Hetherington, 1970). Two astronomers, named Harlow Shapley

and Heber Doust Curtis, argued their views about the Milky Way before the National Academy of Science (NAS) in April 1920. Curtis had studied spiral nebulae and concluded that they are comparable to the Galaxy. On the other hand, using period-luminosity relation of Cepheid variable stars, Shapley had determined the size of the Milky Way and concluded that spiral nebulae are the part of the Galaxy. This is what brought them before the National Academy of Science.

Shapley assumed that the globular clusters (GCs) were part of the Galaxy and considered the maximum extent of the globular clusters to be the size of the Galaxy. To estimate distance of the globular clusters, he assumed that the stars in the other part of the Galaxy and in globular clusters are similar to the nearby stars. Shapley's assumptions led him to conclude that the spiral nebulae had to be at unconvincingly large distances for being as big as the Galaxy (Shapley & Curtis, 1921). Although Shapley provided several arguments to support his estimate for the size of the Galaxy, Curtis was not convinced and raised objections against the assumptions Shapley made. Curtis did not give up his belief that the spiral nebulae were comparable to the Galaxy (Hetherington, 1970).

According to Shapley, the Galaxy was big and the Sun was away from the center of the Galaxy. On the other side, in Curtis' model, the Galaxy was small and the Sun was near the center of the Galaxy. This issue was resolved in 1925, when Edwin Hubble showed that the distance to the M31 spiral nebula was greater than the size of the Galaxy that Shapley estimated. Hubble used the 2.5m Hooker Telescope (which was the biggest telescope of that time) installed at Mount Wilson Observatory to identify Cepheid variable stars and measured the distance to M31. He confirmed that M31 was indeed like the Galaxy (Hubble, 1925). His findings concluded that Shapley was right in estimating the size of our Galaxy and the location of the Sun. In contrast, Curtis correctly identified spiral nebulae as galaxies similar to ours. Nowadays these spiral nebulae are known as galaxies.

1.2   Classification of galaxies

As the number of observed galaxies increased, it became essential to separate them in different groups for their systematic studies. Hubble (1926) presented the first detailed classification of the galaxies based on their structures on the photographic plate. It was latter updated to include other morphological types (de Vaucouleurs, 1959a).

1.2.1 Hubble's classification

Edwin Hubble collected a large sample of galaxies and divided them into two main classes, regular and irregular, primarily based on the morphology of the galaxies on the photographic plates Hubble (1926). The galaxies that show rotation symmetry about their centers were assigned to the regular class. Depending on the flatness of the galaxy and the presence of spiral structure, regular galaxies are divided into two sub-classes: Ellipticals (E) and Spirals. Spiral galaxies are further separated into normal spirals (S) and barred spirals (SB). On the other hand, all the remaining galaxies lacking rotation symmetry are classified as irregular (Irr) galaxies. See Fig. 1.1 for Hubble's classification of the galaxies (Hubble, 1936) in the form of a tuning-fork. It is commonly known as Hubble's tuning-fork diagram.

Fig. 1.1 Hubble's classification of galaxies taken from 'The Realm of the Nebulae'. Ellipticals are arranged along the stem of the fork, and the two sub-classes of spirals are positioned along the prongs of the fork. Furthermore, lenticular galaxies are placed at the base of the fork. Picture courtesy: (Hubble, 1936).

Elliptical galaxies are featureless and show smoothly fading luminosity from center to the outer region. Their projected shapes vary from circular to elongated. The shape of the elliptical is quantified with its ellipticity defined as $\varepsilon = 1 - \frac{b}{a}$, where a and b are the semi-major and semi-minor axis. Ellipticity of the elliptical galaxies falls between 0 (for circular) and 0.7 (for the most elongated). Depending on the ellipticity, an elliptical galaxy is represented by the letter 'En', where $n = 10\varepsilon$. Therefore, circular ellipticals are denoted by E0 and the most elongated is designated as E7. We do not have any elliptical galaxy designated as E8, E9 or E10, because they cannot be dynamically stable.

Spiral galaxies are flat and usually show distinct features such as spiral arms and a central bar. Spirals without the central bar are called normal spirals (S). In contrast, spirals having central bar are known as barred spirals (SB). Spiral galaxies show different level of winding in their spiral arms. Spirals with closely wound and unresolved spiral arms are called early-type galaxies (Sa or SBa), whereas spirals with clearly open and resolved spiral arms are known as late-type spirals (Sc or SBc). In between, there lie the intermediate-type spirals which show relatively open and slightly resolved spiral arms (Sb or SBb). Moving from early-type to late-type galaxies, we see the following smooth transitions: spiral arms start opening, luminosity of the galaxies begins to decrease, gas fraction in the galaxies increases, the relative importance of centrally bright component decreases. Edwin Hubble placed lenticular galaxies (S0) at the transition point from ellipticals to spirals. These galaxies look like the lens, so they are called lenticular galaxies.

1.2.2   de Vaucouleurs' classification

de Vaucouleurs (1959a) took one step ahead of Hubble and updated Hubble's classification to include more refined features seen in spiral galaxies. The following are the changes de Vaucouleurs made in Hubble's classification scheme,

1. de Vaucouleurs used Hubble's designation of ellipticals (E) and barred spirals (SB) as is and renamed normal spirals as SA to include a new class SAB for the galaxies with mixed characteristics (e.g. weakly barred galaxies).

2. To recognize the nuclear structure of the galaxies, he used '(r)' for spirals emerging from ring surrounding the nucleus, '(s)' for spirals emerging from the nucleus, and '(rs)' for the mixed type nucleus.

3. Along with Hubble's early-type ('a'), intermediate-type ('b') and late-type ('c') stages of spiral galaxies, he introduced a very late-type stage ('d') and spiral to irregular transition stage ('m'). In between these main stages, he denoted intermediate stages by 'ab', 'bc', 'cd', etc.

4. He preferred to used '−' and '+' to represent respectively early and late stages of non-spiral galaxies. For example, E^+ shows the late stage of E.

5. Magellanic type irregular galaxies are designated as I(m).

6. To include the information of outer ring like structure seen in spiral galaxies symbol '(R)' is prepended in the main class of the galaxy.

Fig. 1.2 de Vaucouleurs' classification of galaxies taken from 'Classification and Morphology of External Galaxies'. This 3-dimensional representation shows E to I galaxies from left to right, normal spirals at the top, barred spirals at the bottom, ring-shaped (r-shaped) on the far side, and spiral-shaped (s-shaped) on the near side. Two cross-sections at the bottom show the classification system at any location along the axis. Picture courtesy: (de Vaucouleurs, 1959a).

The detailed de Vaucouleurs' classification for galaxies has been shown in Fig. 1.2. It shows E to I galaxies from left to right, normal spirals at the top, barred spirals at the bottom, ring-shaped (r-shaped) on the far side, and spiral-shaped (s-shaped) on the near side. Two cross-sections at the bottom of figure show the classification system at any position along the axis. Fig. 1.3 illustrates the classification scheme using a cross-section near Sb stage of the galaxies.

1.3   Evolution of galaxies

Galaxies are normally found in groups and clusters where they interact with each other gravitationally. Fig. 1.4 shows a cluster of galaxies named SMACS 0723 imaged from Near-Infrared Camera (NIRCam) installed on NASA's James Webb Space Telescope (JWST). It contains thousands of galaxies bound by the huge gravity of the cluster. Gravitational interactions between galaxies plays a crucial role in their formation and evolution over cosmic time. They impact the morphology and kinematics of the galaxies. Interaction of galaxies are categorized in two types: mergers and fly-by interactions.

Fig. 1.3 A visual illustration of de Vaucouleurs' classification using a cross-section near Sb stage of the galaxies. See Section 1.2.2 for the nomenclature scheme shown in this figure. Picture courtesy (de Vaucouleurs, 1959a).

- **Mergers:** Mergers are events where two or more galaxies come closer as result of their tremendous gravitational attraction and fall into each other. Mergers are also called collisions of galaxies. These interactions are very violent and affect galaxies very significantly. Depending on the mass ratio of two merging galaxies, mergers can be classified into major mergers and minor mergers. Suppose, M_p is the mass of primary (high-mass) galaxy and M_s is the mass of secondary (low-mass) galaxy in a merger event. In general, mergers with M_p:$M_s \leq 3$:1 are called major mergers, and the mergers with M_p:$M_s > 3$:1 are termed minor mergers.

- **Fly-bys:** In contrast to the mergers, fly-bys are events where two or more galaxies come closer, exerting enormous gravitation pull on each other, and then separate without merging. Fly-bys are commonly referred to as tidal encounters or interactions. Similar to mergers, fly-bys can also be categorized into major flybys and minor flybys depending on the mass ratio of interacting galaxies. For example, M_p is the mass of the primary (high-mass) galaxy and M_s is the mass of the secondary (low-mass) galaxy in a flyby

Fig. 1.4 A cluster of galaxies, SMACS 0723, as seen from Near-Infrared Camera (NIRCam) instrument onboard NASA's James Webb Space Telescope (JWST). It shows thousands of galaxies in a tiny part of the sky represented by a sand grain on arm-length. The arc-shaped structures are more distant galaxies behind the cluster which are magnified due to enormous gravity of the cluster. Objects with sharp spikes are foreground stars. Picture courtesy NASA, ESA, CSA, and STScI.

event. Then the fly-bys with M_p:$M_s \leq 3:1$ are called major fly-bys, and the fly-bys with M_p:$M_s > 3:1$ are termed minor fly-bys.

Two interacting galaxies can also go through several fly-by events prior to the final merger event. It mainly depends on the initial orbits of the interacting galaxies. If both the galaxies approach radially towards each other, they will directly collide and merge. On the other hand, if both the galaxies approach on elliptical orbits, they will keep flying past each other until their orbits decay significantly (due to dynamical friction).

Cosmological simulations suggest that mergers were more frequent at higher redshifts compared to fly-by interactions. As the Universe evolved and expanded, the frequency of fly-by interactions increased and become comparable to mergers (Yee & Ellingson, 1995; Sinha & Holley-Bockelmann, 2012a; An et al., 2019a). Both types of interactions are important in the formation and evolution of the galaxies (Di Matteo et al., 2007; Li et al., 2008; Kim et al., 2014a; Sinha & Holley-Bockelmann, 2015a). Both interactions can induce enhanced

star formation, extended tidal features, spiral arms, bars, etc. The results of these interactions depend on various factors such as their initial mass ratio, gas contents, orbital configurations and distance of closest approach (pericenter distance).

1.4 Major components of galaxies

Elliptical galaxies are featureless and fade smoothly as we move from center to the outer edge. Their one dimensional radial surface brightness along the isophotal semi-major axis R is well represented with Sérsic profile (Sersic, 1968) defined as

$$\Sigma(R) = \Sigma_e \exp\left[-b_n\left\{\left(\frac{R}{R_e}\right)^{\frac{1}{n}} - 1\right\}\right], \tag{1.1}$$

where R_e is the effective radius or half-light radius, Σ_e is the surface brightness at R_e, and b_n is the function of Sérsic index 'n' and is related with Gamma function (Γ) and lower incomplete Gamma function (γ) by the following expression,

$$\gamma(2n; b_n) = \frac{1}{2}\Gamma(2n), \tag{1.2}$$

For 1<n<10, b_n cab be approximated as $b_n = 2n - 1/3$ (Ciotti & Bertin, 1999). Sérsic index ranges from n = 2 for dim elliptical galaxies to m = 6 for luminous elliptical galaxies. In the middle, n = 4 gives de Vaucouleurs' $R^{1/4}$ profile (de Vaucouleurs, 1948).

Elliptical galaxies contain very little or no gas and dust. They are generally red in color (Sandage & Visvanathan, 1978), which is an indication of a dominant older stellar population. All or a major fraction of the stars in elliptical galaxies orbit randomly around the center (Bertola & Capaccioli, 1975; Illingworth, 1977). The stellar velocity dispersion supports them against gravitational collapse. The fact that elliptical galaxies are dominated by dispersion suggests that they have formed in violent processes. Several theoretical, numerical, and observational works have shown that elliptical galaxies are formed in monolithic collapses or/and in similar mass mergers (Bekki & Shioya, 1997; Zepf, 1997; De Lucia et al., 2006; Bournaud et al., 2007b; Kormendy et al., 2009; Naab et al., 2014).

In contrast, spiral galaxies possess various distinct features such as spiral arms, bars, and bulges. We refer to these structures as the components of (spiral) galaxies. In the following subsections, we give a brief overview of the major galactic components.

1.4.1   Disk

Most of the baryonic mass of spiral galaxies lies in the form of a flat circular disk. The radial distribution of surface brightness of this disk is well described with an exponential profile (Freeman, 1970) defined as

$$\Sigma(R) = \Sigma_0 \exp\left(-\frac{R}{R_\mathrm{s}}\right), \tag{1.3}$$

where Σ_0 is the central surface brightness of the disk and R_s is the disk scale radius (or scale length). In many cases, due to the presence of other central components, the central surface brightness of the disk is extrapolated to the outer disk during the fitting process. Additionally, the vertical surface brightness of the disk is commonly approximated with a sech^2 function. Therefore, the three dimensional (3D) disk surface brightness profile is given by the following expression,

$$j(R,z) = j_0 \exp\left(-\frac{R}{R_\mathrm{s}}\right) \mathrm{sech}^2\left(\frac{z}{z_0}\right), \tag{1.4}$$

where j_0 is the central surface brightness at the mid-plane of the disk, R_s is disk scale radius, and z_0 is disk scale height. The disk scale height is nearly constant at each radial position for late type spirals. However, early type galaxies show increasing disk scale height with radius (de Grijs & Peletier, 1997).

The majority of stars in spiral galaxies rotate around the galactic center on near circular orbits. The rotation of stars in spiral galaxies provides support against gravitational collapse of the disk (Mo et al., 2010). It is still not fully understood how spiral galaxies develop disks and whether they retain them throughout the cosmic evolution. In Λ Cold Dark Matter (ΛCDM) models of galaxy formation[1], disks form in dark matter halos that have high angular momentum (Dalcanton et al., 1997).

1.4.2   Spiral arms

As the name suggests, spiral galaxies host beautiful spiral arms in their disks. The extent of the spiral arms ranges from the very central region to the edge of the galaxy. The strength of spiral arms is often quantified using the Fourier decomposition of disk surface brightness. For example, in a polar co-ordinate system centered at the nucleus of the galaxy, $\Sigma(\mathrm{R}, \phi)$ is the surface brightness of a spiral galaxy at any co-ordinate (R, ϕ) and m is the number of spiral arms. Then its surface brightness can be written as follows,

$$\Sigma(\mathrm{R}, \phi) = \Sigma_0(\mathrm{R}) + \Sigma_\mathrm{m}(\mathrm{R}) \cos\left[\mathrm{m}\phi + \phi_\mathrm{m}(\mathrm{R})\right], \tag{1.5}$$

---

[1] also known as hierarchical structure formation model.

where $\Sigma_0(R)$ and $\Sigma_m(R)$ are respectively amplitudes of the $0th$ and mth Fourier modes at any radius R. $\phi_m(R)$ is the phase angle of the mth Fourier mode. Now, the ratio $\Sigma_m(R)/\Sigma_0(R)$ provides a quantitative strength of an m-armed spiral pattern present in the disk at any radial position R.

The shape of spiral arms is reasonably well represented by a logarithmic profile. The mathematical form of logarithmic spiral arms is given by the following expression,

$$R(\phi) = R_0 e^{\phi \tan \alpha}, \tag{1.6}$$

where $R_0 = R(0)$, ϕ is the winding angle which constraints the azimuthal extent of the spiral arm, and α is the pitch angle which measures the tightness of the spiral arm. For grand design spirals[2], pitch angle usually remains constant throughout the galaxy within an error of ± 5 degree (Kennicutt, 1981; Díaz-García et al., 2019).

The enhanced star formation in disk galaxies is often seen along their spiral arms which indicates that the over-densities associated with spiral arms act as the sites for star formation. They sweep the inter stellar medium and help in the birth of new stars. Spiral arms also help transferring angular momentum from the inner region to the outer region (Binney & Tremaine, 2008a). This angular momentum transfer is useful for the growth of central disk. Furthermore, Spiral arms are thought to be source of wave-like breathing motion seen in the Milky Way (Kumar et al., 2022c).

There are various hypothesis for the formation of spiral arm. However, the fundamental reason for their existence is that the spiral galaxies are dynamically cold stellar systems[3] and rotate differentially.

1.4.3 Bar

Bars are narrow, linear structures observed across the centers of spiral galaxies. As discussed in Section 1.2, spiral galaxies with bars are known as barred spirals. Nearly two-third of spirals in the local volume are barred galaxies. Similar to the spiral arms, bars can also be quantified using Fourier decomposition of the disk surface brightness. Since bars have 2-fold symmetry i.e. in one full rotation disk surface brightness looks unchanged twice, we can replace m by 2 in equation (1.5). Now, the surface brightness of barred spirals can be expressed as follows,

$$\Sigma(R, \phi) = \Sigma_0(R) + \Sigma_2(R) \cos\left[2\phi + \phi_2(R)\right], \tag{1.7}$$

---

[2]Grand design spirals is a class of spiral arms that shows two-armed spiral pattern clearly traceable to large radial extent.

[3]In astronomy, by dynamically cold stellar systems, we mean that these systems are rotation dominating. Conversely, dispersion supported stellar systems are called dynamically hot.

where $\phi_2(R)$ is the phase angle of *2nd* Fourier mode. It remains constant in bar region because bars are linear structures. Now, the ratio $\Sigma_2(R)/\Sigma_0(R)$ provides quantitative strength of bar at any radial position R.

Bars play an important role in the secular evolution of spiral galaxies. Bars can shock the gas and ignite star formation in the inner disk region. Funneling of gas along the bar and into the galaxy nucleus helps the growth of central super massive black holes (SMBHs).

1.4.4   Bulge

The dense collection of stars in the center of disk galaxies is knows as the bulge. Moreover, galaxies which do not have any bulge are called bulgeless galaxies. On the basis of morphology and the dynamics of its constituent stars, bulges can be categorized into two types: classical bulges and pseudo-bulges. Classical bulges are nearly spherical in shape and are supported by the rotational motion of the stars, whereas pseudo-bulges are flat structures and show rotational support (Kormendy et al., 2006a; Drory & Fisher, 2007a; Athanassoula, 2008; Fisher & Drory, 2008a,d). In Fig. 1.5, we have shown different types of bulges as seen in observed galaxies.

There are many observational similarities between classical bulges and elliptical galaxies which suggests a similar formation mechanism for both systems. For example, their shapes or Sérsic indices, the orbit families of their stars, their size-luminosity relation, etc (Faber & Jackson, 1976; Kormendy, 1977; Gültekin et al., 2009; Savorgnan et al., 2013). On the basis of numerical simulations and theoretical studies, it is well accepted that elliptical galaxies are the remnants of galaxy mergers (Balland et al., 1998; Dubinski, 1998; Burkert & Naab, 2003; Bournaud et al., 2007c). Classical bulges are also thought to form in monolithic collapse of gas clouds, and later grow from the accretion of smaller galaxies (Aguerri et al., 2001a; Bournaud et al., 2005a; Brooks & Christensen, 2016).

The term pseudo-bulge is commonly used to describe two types of bulges: disky pseudo-bulges and boxy/peanut/x-shape pseudo-bulges. Disky pseudo-bulges are circular in shape but in the vertical direction they are as flat as the disks of their host galaxy. Hence, it is nearly impossible to detect them in edge-on galaxies. They are thought to form within the inner disks via star formation (Laurikainen et al., 2009). Boxy/peanut/x-shape bulges (BPX bulges in short) are more extended in the vertical direction and can hence be detected in edge-on galaxies. They are usually associated with bars in disk galaxies (Friedli & Pfenniger, 1990; Debattista et al., 2006; Gadotti, 2011). Pseudo-bulges, unlike classical bulges, are formed due to the secular evolution of galactic disks, the buckling instability of bars (Combes & Sanders, 1981), or via mergers with gas rich galaxies (Keselman & Nusser, 2012).

Fig. 1.5 Few examples of different types of bulges. Name of the galaxy and the class of its bulge are mentioned in corresponding panels. Picture courtesy: this collage is made using images from NASA/ESA/Hubble and from Aladin Lite's SDSS9.

1.4.5 Dark matter halo

Dark matter is an unknown form of the matter and accounts for more than 80% of the matter in galaxies. It is totally invisible as it does not emit electromagnetic radiation. We have only indirect evidences that dark matter exists from its gravitational interaction with visible baryonic matter. There have been several attempts to detect dark matter and discover its nature but to date no one has succeeded.

Zwicky (1933) was the first who quantitatively showed the presence of dark matter. He calculated the mass of Coma galaxy cluster using kinematics of galaxies within the cluster. Surprisingly, he found that the mass estimated using kinematics is nearly 400 times the mass

estimated using the luminosity of all galaxies in the cluster. Later on, Freeman (1970) and
Rubin & Ford (1970) independently showed that the rotation curves[4] of spiral galaxies become
flat in the outer regions (see Fig. 1.6 for a typical rotation curve). The central rotation curve
can be explained with baryonic matter, while the outer rotation curve is hard to explain without
assuming extra hidden matter i.e. dark matter. These evidences suggested the need of dark
matter in the Universe (e.g., see review on dark matter de Swart et al., 2017).

Fig. 1.6 A typical example of rotation curve. Circular velocity increases up to certain radius
and then it becomes constant. Here, solid curves show fitted rotation curve and contribution of
individual component. Picture courtsey: (van Albada et al., 1985).

Dark matter provides gravitational support for the formation of the structure we see today.
Early clustering of dark matter results in dark matter halo formation. The huge gravitational
potential of these halos helps gas in-flow toward their center, and leads to the formation of
proto-galaxies. The merger of small dark matter halos and continuing star formation help the
growth of galaxies. Dark matter stabilizes disk galaxies against global instabilities such as bars
and spirals.

1.5   Our galaxy: The Milky Way

The Milky Way is a barred spiral galaxy comprising of a disk and a BPX bulge (Ness & Lang,
2016). The radius of the Milky Way disk is around 10 kpc which contains most of its stars

---

[4]Rotation curve is the radial variation of orbital velocity as a function of radius from galactic center.

Fig. 1.7 Top: the artistic face-on view of the Milky Way. Two main spiral arms, Perseus and Scutum-Centaurus, are originating from ends of the central bar. Picture courtesy: NASA/JPL-Caltech/R. Hurt (SSC/Caltech). Bottom: the edge-on view of the Milky Way taken from Diffuse Infrared Background Experiment (DIRBE) instrument of Cosmic Background Explorer (COBE) satellite. Picture courtesy: Ned Wright.

(Binney & Tremaine, 2008a). The top and bottom panels of Fig. 1.7 show the face-on and edge-on view of the Milky Way respectively. Face-on view is the detailed artistic impression of the Galaxy. It shows the central bar and the spiral arms in the disk. Edge-on view shows the extent of disk and dense collection of stars in the central region. This image is observed with Diffuse Infrared Background Experiment (DIRBE) instrument onboard Cosmic Background Explorer (COBE) satellite. Here, we cannot clearly distinguish X-shape of the bulge due to our

position in the Galaxy. The long axis of the bar makes $\sim 30°$ angle from our line-of-sight to the Galactic center (Wegg et al., 2015).

In total, the Milky Way weight is equivalent to 1.2 trillion times the Sun's mass (e.g., see Wang et al., 2020a). The stellar mass of the Milky way is $\sim 6 \times 10^{10} M_\odot$ (where $M_\odot =$ solar mass) which includes a disk of $\sim 5 \times 10^{10} M_\odot$ and a bulge of $\sim 1 \times 10^{10} M_\odot$ (McMillan, 2011; Licquia & Newman, 2015). The scale length of the disk is nearly 3 kpc and the half length of bar is 4 to 5 kpc (Cabrera-Lavers et al., 2008; McMillan, 2011; Wegg et al., 2015). The extent of its dark matter halo is not well constrained and expected to lie between 100 and 300 kpc (Dehnen et al., 2006; McMillan, 2011; Deason et al., 2020). The Sun is located at a distance of 8.3 kpc from the center of the Milky Way and completes one orbit around Galactic center every 240 million years at an average speed of 220 km s^{-1} (McMillan, 2011; Gillessen et al., 2017).

The Milky Way is the only galaxy studied in great detail by far. Still, there is a lot to know about it. Recent studies based on enormous 6D data (3 position and 3 velocity) have shown that the Milky Way disk is not perfectly planar. It exhibits a warped disk (Freudenreich et al., 1994; Poggio et al., 2018), north-south star number asymmetry and wave-like breathing motion (Widrow et al., 2012; Kumar et al., 2022c), corrugation or undulation rings (Xu et al., 2015), warp in solar neighbourhood (Schönrich & Dehnen, 2018), phase-space spirals (Antoja et al., 2018a).

1.6 Thesis layout

Chapter 1 reviews history of galaxies, their classification, evolution, galactic components, and the Milky Way. Two-body relaxation, theoretical modelling of stellar orbits, collisionless Botlzmann equation, Jeans equations, numerical techniques, simulations, and codes used for this thesis are discussed in chapter 2. We have presented our work on the evolution of bulges, disks, and spiral arms in fly-by interactions of galaxies in chapter 3. In the chapter 4, we have discussed the excitation of wave-like breathing motion by tidally induced spiral arms in fly-by interactions. Chapter 5 of this thesis talks about the formation of bar and BPX bulges in non-spherical dark matter halos. Chapter 6 presents our work on the growth of disk-like pseudo-bulges in observed galaxies since $z = 0.1$ redshift. Next, chapter 7 shows our work on bulgeless galaxies in Illustris TNG50 simulations and comparison with local volume observations. Finally, chapter 8 concludes our work in this thesis along with future directions.

2
Techniques for Numerical Simulations

2.1 Introduction

A significant fraction of the baryonic mass of galaxies lies in the form of stars. The orbits of these stars characterize the morphology, shape and internal kinematics of galaxies. For example, the random stellar orbits in elliptical galaxies make them ellipsoidal, whereas the ordered rotational stellar orbits in spiral galaxies make them resemble a disk. Therefore, to understand the formation and evolution of galaxies, one needs to study how the stars move in galaxies. However, it is challenging to examine stellar orbits analytically, given the large number of stars in galaxies or in other stellar systems. Thus stellar dynamics in galaxies is complex and is mainly investigated using a variety of approximations and numerical techniques. In this chapter we review some analytical and numerical approaches to study the stellar kinematics in galaxies.

2.2 Two body relaxation

Several theoretical studies of galaxies are based on the smooth potential of galaxies. This is a reasonable assumption and is better understood via two-body relaxation processes. Consider

Fig. 2.1 A toy model to measure the effect of an encounter on a subject star assuming a straight line trajectory and impact parameter b. Picture courtesy: (Binney & Tremaine, 2008a).

the motion of a subject star in a galaxy having N number of identical stars each with mass m. Say, the subject star is moving with velocity $\vec{v}$ and passes-by a field star at a distance b (known as the impact parameter). For simplicity, we assume that the field star is stationary during this interaction and the trajectory of a subject star is a straight line (see Fig. 2.1). In this case, the net force on the subject star along the trajectory will sum up to zero. However, the force perpendicular to the trajectory at any instance will be given by

$$F_\perp = \frac{Gm^2}{r^2}\cos\theta = \frac{Gm^2}{r^2}\frac{b}{r} = \frac{Gm^2 b}{r^3},$$
$$F_\perp = \frac{Gm^2 b}{(b^2 + x^2)^{3/2}}. \tag{2.1}$$

Let us assume, time is zero at the closest passage. Now using Newton's law in perpendicular direction,

$$m\frac{dv_\perp}{dt} = F_\perp$$
$$\int m dv_\perp = \int_{-\infty}^{+\infty} F_\perp dt$$
$$m\delta v_\perp = \int_{-\infty}^{+\infty} F_\perp dt = \int_{-\infty}^{+\infty} \frac{Gm^2 b}{(b^2 + x^2)^{3/2}} dt$$
$$m\delta v_\perp = \int_{-\infty}^{+\infty} \frac{Gm^2 b}{(b^2 + x^2)^{3/2}} \frac{dx}{v}$$
$$m\delta v_\perp = \frac{Gm^2 b}{v} \int_{-\infty}^{+\infty} \frac{dx}{(b^2 + x^2)^{3/2}}$$

$$m\delta v_\perp = \frac{Gm^2 b}{v}\frac{2}{b^2}$$
$$\delta v_\perp = \frac{2Gm}{bv}. \tag{2.2}$$

This is the magnitude of deflection in a subject star's velocity due to a single encounter with a field star i.e. $|\delta\vec{v}| = \delta v = \delta v_\perp$. For $90°$ deflection ($\delta v = v$), impact parameter will be $b_{90} = 2Gm/v^2$.

Suppose R is the size of the galaxy, its number surface density will be of the order of $N/\pi R^2$. Therefore, the number of encounters per crossing in the impact parameter range from b to $b + db$ can be written as

$$\delta n \approx \frac{N}{\pi R^2} 2\pi b db = \frac{2N}{R^2} b db. \tag{2.3}$$

All these encounters will change a subject star's velocity randomly and result in zero net deflection. However, the sum of the squared deflections will be non-zero and can be estimated by integrating over whole range of impact parameter,

$$(\Delta v)^2 \approx \int_{b_{min}}^{b_{max}} (\delta v)^2 \delta n$$
$$(\Delta v)^2 \approx \int_{b_{min}}^{b_{max}} \left(\frac{2Gm}{bv}\right)^2 \frac{2N}{R^2} b db$$
$$(\Delta v)^2 \approx 8N \left(\frac{Gm}{Rv}\right)^2 \ln\left(\frac{b_{max}}{b_{min}}\right),$$
$$(\Delta v)^2 \approx 8N \left(\frac{Gm}{Rv}\right)^2 \ln\Lambda, \tag{2.4}$$

where

$$\ln\Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right), \tag{2.5}$$

is the Coulomb logarithm. It can be approximated assuming $b_{min} = b_{90}$ and $b_{max} = R$ for most of the stellar systems where $R >> b_{90}$. Now,

$$\frac{(\Delta v)^2}{v^2} \approx 8N \left(\frac{Gm}{Rv^2}\right)^2 \ln\Lambda. \tag{2.6}$$

We can eliminate R from this equation considering the typical speed of a star in a galaxy to be equal to the circular velocity at the edge of the galaxy i.e. $v^2 = GNm/R$.

$$\frac{(\Delta v)^2}{v^2} \approx \frac{8 \ln \Lambda}{N}. \tag{2.7}$$

Every time a subject star crosses the galaxy, its velocity changes by $(\Delta v)^2$. Using this expression, we can calculate number of crossings a subject star will make to change its velocity by the same order. In other words, the number of required crossings to make the star dynamically relax is given by,

$$n_{relax} \approx \frac{N}{8 \ln \Lambda}, \tag{2.8}$$

and the relaxation time $t_{relax} = n_{relax} t_{cross}$, where $t_{cross} = R/v$ is the galaxy crossing time. Furthermore, $\Lambda \approx R/b_{90} \approx Rv^2/Gm = N$. Therefore,

$$t_{relax} \approx \frac{0.1N}{\ln N} t_{cross}. \tag{2.9}$$

Relaxation time quantifies when the cumulative effect of small encounters significantly changes the orbit of a star. After one relaxation time, a star does not remember its initial conditions. Thus, for an evolution time $t < t_{relax}$, we can consider a system to be collisionless. Also, it is reasonable to neglect the effect of the discreteness of the gravitational potential due to individual stars and consider the potential to be smooth.

2.3 Stellar Orbits in spherical potential

The simplest approximation to study stellar orbit is the spherically symmetric potential of a stellar system. In a spherical potential, the star's angular momentum is a conserved quantity, meaning the star orbits in a plane. Let us assume a polar co-ordinate system (r, ψ) in the orbital plane positioned at the center of potential $\Phi(r)$. Now, we can write its Lagrangian per unit mass as follows,

$$\mathscr{L} = \frac{1}{2} \left[\dot{r}^2 + (r\dot{\psi})^2 \right] - \Phi(r). \tag{2.10}$$

Equations of motion for this Lagrangian are,

$$\frac{d}{dt} \frac{\partial \mathscr{L}}{\partial \dot{r}} - \frac{\partial \mathscr{L}}{\partial r} = 0 \implies \ddot{r} - r\dot{\psi}^2 + \frac{d\Phi}{dr} = 0, \tag{2.11}$$

$$\frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\psi}} - \frac{\partial \mathscr{L}}{\partial \psi} = 0 \implies \frac{d}{dt}(r^2 \dot{\psi}) = 0. \tag{2.12}$$

Equation (2.12) shows that $r^2\dot{\psi}$ is a constant of motion which is nothing but the angular momentum (L) of the star i.e.,

$$r^2\dot{\psi} = L. \tag{2.13}$$

This equation can be re-arranged in the form of an operator,

$$\frac{d}{dt} = \frac{L}{r^2}\frac{d}{d\psi} \tag{2.14}$$

Using this operator, we can replace the time derivative by an angle derivative in the equation (2.11) as follows,

$$\frac{L}{r^2}\frac{d}{d\psi}\left(\frac{L}{r^2}\frac{dr}{d\psi}\right) - r\left(\frac{L}{r^2}\right)^2 + \frac{d\Phi}{dr} = 0,$$
$$\frac{L^2}{r^2}\frac{d}{d\psi}\left(\frac{1}{r^2}\frac{dr}{d\psi}\right) - \frac{L^2}{r^3} + \frac{d\Phi}{dr} = 0. \tag{2.15}$$

We can simplify equation (2.15) by substituting,

$$r = \frac{1}{u} \implies \frac{dr}{d\psi} = -\frac{1}{u^2}\frac{du}{d\psi}. \tag{2.16}$$

On substitution of these values, equation (2.15) becomes,

$$-L^2u^2\frac{d^2u}{d\psi^2} - L^2u^3 + \frac{d\Phi}{dr} = 0,$$
$$\frac{d^2u}{d\psi^2} + u = \frac{1}{L^2u^2}\frac{d\Phi}{dr}. \tag{2.17}$$

To integrate equation (2.17), multiply both sides by $du/d\psi$, and simplify the right-hand side using equation (2.16),

$$\frac{du}{d\psi}\frac{d^2u}{d\psi^2} + u\frac{du}{d\psi} = -\frac{1}{L^2}\frac{d\Phi}{d\psi},$$
$$\frac{1}{2}\left(\frac{du}{d\psi}\right)^2 + \frac{1}{2}u^2 = -\frac{\Phi}{L^2} + \frac{E}{L^2},$$
$$\left(\frac{du}{d\psi}\right)^2 + u^2 + \frac{2\Phi}{L^2} = \frac{2E}{L^2}, \tag{2.18}$$

where E/L^2 is an integration constant and E represents total energy of the orbit. Now, we can write equation (2.18) in polar co-ordinates using equations (2.14) and (2.16) as follows,

$$\frac{1}{L^2}\left(\frac{dr}{dt}\right)^2 + \frac{1}{r^2} + \frac{2\Phi}{L^2} = \frac{2E}{L^2},$$

$$\left(\frac{dr}{dt}\right)^2 = 2(E - \Phi) - \frac{L^2}{r^2}, \tag{2.19}$$

Using equation (2.19), we can estimate orbital parameters of the stellar orbit. For example,

Fig. 2.2 An example orbit in a spherical potential. It looks like a rosette which is typical in spherical potential. Picture courtesy: (Binney & Tremaine, 2008a).

- **Pericenter and apocenter distances:** From the center of potential, the closest point of orbit is known as perigee and the farthest point is called apogee. Radial distances of perigee and apogee are respectively labeled as pericenter (r_p) and apocenter (r_a) distances. At perigee and apogee of the orbit, dr/dt will be zero i.e.,

$$0 = 2(E - \Phi) - \frac{L^2}{r^2},$$

$$r^2 - \frac{L^2}{2(E - \Phi)} = 0. \tag{2.20}$$

This equation generally has two roots r_p and r_a. A star oscillates between these two radii as it evolves.

- **Radial period** (T_r)**:** It is the time required to orbit from perigee to apogee and back to perigee. Thus,

$$T_r = 2 \int_{r_p}^{r_a} \frac{dt}{dr} dr,$$

$$T_r = 2 \int_{r_p}^{r_a} \frac{dr}{\sqrt{2\left(E - \Phi\right) - \frac{L^2}{r^2}}} \tag{2.21}$$

- **Azimuthal period** (T_ψ)**:** It is the time required to travel 2π radian in azimuthal direction. Let us first calculate the change in azimuthal angle in one radial period.

$$\Delta\psi = 2 \int_{r_p}^{r_a} \frac{d\psi}{dr} dr = 2 \int_{r_p}^{r_a} \frac{L}{r^2} \frac{dt}{dr} dr,$$

$$\Delta\psi = 2L \int_{r_p}^{r_a} \frac{dr}{r^2 \sqrt{2\left(E - \Phi\right) - \frac{L^2}{r^2}}} \tag{2.22}$$

Now, azimuthal period can be calculated as,

$$T_\psi = \left|\frac{2\pi}{\Delta\psi}\right| T_r \tag{2.23}$$

For a close orbit, $\Delta\psi/2\pi$ should be a rational number which it is not generally. Therefore, the typical orbits in a spherical potential describe a rosette like structure, as can be seen in Fig. 2.2.

2.4   Stellar Orbits in an axisymmetric potential

The assumption of a spherically symmetric potential completely breaks down for disk galaxies. Thus, disk galaxies are studied considering an axisymmetric potential. Stars moving in the equatorial plane of such a potential feel zero net force in the vertical direction. As a consequence of this the star cannot perceive that it is not moving in a spherical potential. Therefore, stellar orbits in the equatorial plane are described in a similar manner as we discussed in Section 2.3.

To study typical stellar orbits in axisymmetric potential $\Phi(R, z)$ which is symmetric about $z = 0$ plane, let us consider a cylindrical co-ordinate system (R, ϕ, z) positioned at the center of the potential. Now, we can write its Lagrangian per unit mass as follows,

$$\mathscr{L} = \frac{1}{2}\left[\dot{R}^2 + (R\dot{\phi})^2 + \dot{z}^2\right] - \Phi. \tag{2.24}$$

The momenta for this Lagrangian are,

$$p_R = \frac{\partial \mathscr{L}}{\partial \dot{R}} = \dot{R} \tag{2.25a}$$

$$p_\phi = \frac{\partial \mathscr{L}}{\partial \dot{\phi}} = R^2 \dot{\phi} \tag{2.25b}$$

$$p_z = \frac{\partial \mathscr{L}}{\partial \dot{z}} = \dot{z}. \tag{2.25c}$$

The corresponding Hamiltonian is,

$$\mathscr{H} = \frac{1}{2} \left[p_R^2 + \frac{p_\phi^2}{R^2} + p_z^2 \right] + \Phi. \tag{2.26}$$

The equation of motion for this Hamiltonian are,

$$\dot{p}_R = \ddot{R} = -\frac{\partial \mathscr{H}}{\partial \dot{R}} = \frac{p_\phi^2}{R^3} - \frac{\partial \Phi}{\partial R} \tag{2.27a}$$

$$\dot{p}_\phi = \frac{d}{dt}(R^2 \dot{\phi}) = -\frac{\partial \mathscr{H}}{\partial \dot{\phi}} = 0 \tag{2.27b}$$

$$\dot{p}_z = \ddot{z} = -\frac{\partial \mathscr{H}}{\partial \dot{z}} = -\frac{\partial \Phi}{\partial z}. \tag{2.27c}$$

Equation (2.27b) shows that p_ϕ is a conserved quantity and is, in fact, the component of angular momentum about the z-axis (L_z), i.e.

$$p_\phi = R^2 \dot{\phi} = L_z. \tag{2.28}$$

Re-writing equations (2.27a) and (2.27c) using value of $p_\phi = L_z$,

$$\ddot{R} = -\frac{\partial \Phi_{eff}}{\partial R} \tag{2.29a}$$

$$\ddot{z} = -\frac{\partial \Phi_{eff}}{\partial z}, \tag{2.29b}$$

where

$$\Phi_{eff} = \frac{L_z^2}{2R^2} + \Phi(R, z). \tag{2.29c}$$

is effective potential. Coupled equations (2.29a) and (2.29b) describe motion of the star in the $R - z$ plane (also known as the meridional plane). These equations can be solved numerically (or analytically for some special potentials) for different initial conditions. In Fig. 2.3, we have

shown the orbits of a star in the meridional plane of an axisymmetric potential given by the following expression,

$$\Phi(R,z) = \frac{1}{2}v_0^2 \ln\left(R^2 + \frac{z^2}{q^2}\right).$$

(2.30)

Both the orbits are calculated for the same total energy ($E = -0.8$) and the same angular momentum along the z-axis ($L_z = 0.2$) in this potential for $q = 0.9$ and $v_0 = 1$

Fig. 2.3 Example orbits in an axisymmetric potential. Both the orbits are shown for the same total energy and the same angular momentum along z-axis in the potential given in equation (2.30) for $q = 0.9$ and $v_0 = 1$. Picture courtesy: (Binney & Tremaine, 2008a).

2.5   Collisionless Boltzmann Equation

In previous sections, we saw that galaxies are collisionless stellar systems and their potentials can be smoothed to study stellar orbits. However, it is not practical and worthwhile to follow the orbits of each star in a galaxy. For practical purpose, we use a distribution function (DF) $f(\vec{x},\vec{v},t)$ to describe galaxies at any given time t. Here, $f(\vec{x},\vec{v},t)$ is a phase-space probability density function, where the probability of finding a random star at $\vec{w} = (\vec{x},\vec{v})$ in volume dV at any time t is given by,

$$p = \int_{dV} f(\vec{w},t)d^6\vec{w} = \int_{dV} f(\vec{x},\vec{v},t)d^3\vec{x}d^3\vec{v}.$$

(2.31)

By the definition of probability density function,

$$\int_V f(\vec{x},\vec{v},t)d^3\vec{x}d^3\vec{v} = 1.$$

(2.32)

the DF of the system changes over time as the stars travel through phase-space. Analogous to the continuity equation for mass conservation in fluid dynamics, we can write continuity equation for probability conservation in phase-space as follows,

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \vec{w}} \cdot (f\dot{\vec{w}}) = 0 \tag{2.33}$$

Let us say $\mathcal{H}$ is the Hamiltonian in phase-space $\vec{w} = (\vec{x}, \vec{v})$,

$$\begin{aligned}
\frac{\partial}{\partial \vec{w}} \cdot (f\dot{\vec{w}}) &= \frac{\partial}{\partial \vec{x}} \cdot (f\dot{\vec{x}}) + \frac{\partial}{\partial \vec{v}} \cdot (f\dot{\vec{v}}) \\
&= \frac{\partial}{\partial \vec{x}} \cdot (f\frac{\partial \mathcal{H}}{\partial \vec{v}}) - \frac{\partial}{\partial \vec{v}} \cdot (f\frac{\partial \mathcal{H}}{\partial \vec{x}}) \\
&= \frac{\partial f}{\partial \vec{x}} \cdot \frac{\partial \mathcal{H}}{\partial \vec{v}} - \frac{\partial f}{\partial \vec{v}} \cdot \frac{\partial \mathcal{H}}{\partial \vec{x}} \\
&= \frac{\partial f}{\partial \vec{x}} \cdot \dot{\vec{x}} + \frac{\partial f}{\partial \vec{v}} \cdot \dot{\vec{v}}
\end{aligned} \tag{2.34}$$

Substituting the value of equations (2.34) in equation (2.33) gives,

$$\frac{\partial f}{\partial t} + \dot{\vec{x}} \cdot \frac{\partial f}{\partial \vec{x}} + \dot{\vec{v}} \cdot \frac{\partial f}{\partial \vec{v}} = 0. \tag{2.35}$$

This partial differential equation is knows as collisionless Boltzmann equation (CBE) or Vlasov equation. The left-hand side of equation (2.35) is actually a perfect time derivative of f. Thus, simply $df/dt = 0$ is the collisionless Boltzmann equation.

The collisionless Boltzmann equation is based on the conservation of probability density function. We can modify it for the creation and destruction of stars in a galaxy as follows,

$$\frac{\partial f}{\partial t} + \dot{\vec{x}} \cdot \frac{\partial f}{\partial \vec{x}} + \dot{\vec{v}} \cdot \frac{\partial f}{\partial \vec{v}} = C - D, \tag{2.36}$$

where $C(\vec{x}, \vec{v}, t)$ and $D(\vec{x}, \vec{v}, t)$ are respectively creation and destruction rates of stars per unit phase-space volume.

2.6   Jeans equations

One can calculate various observational quantities (velocity moments) using the DF. Nonetheless, finding the DF is not that simple. Here, we review how to estimate various velocity moments using the gravitational potential of the system. Let us say $\Phi(\vec{x})$ is the potential of

galaxy, we can the write collisionless Boltzmann equation (2.35) as follows,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \frac{\partial \Phi}{\partial \vec{x}} \cdot \frac{\partial f}{\partial \vec{v}} = 0. \tag{2.37}$$

Integrating this equation over the whole velocity space and writing it in a summation convention [1],

$$\int \frac{\partial f}{\partial t} d^3\vec{v} + \int v_i \frac{\partial f}{\partial x_i} d^3\vec{v} - \int \frac{\partial \Phi}{\partial x_i} \frac{\partial f}{\partial v_i} d^3\vec{v} = 0,$$
$$\int \frac{\partial f}{\partial t} d^3\vec{v} + \int v_i \frac{\partial f}{\partial x_i} d^3\vec{v} - \frac{\partial \Phi}{\partial x_i} \int \frac{\partial f}{\partial v_i} d^3\vec{v} = 0. \tag{2.38}$$

Since f is the function of $\vec{x}$, $\vec{v}$, and t, the partial derivatives in the first and second terms can be taken outside. The volume integration in the third term can be converted into surface integration using divergence theorem. At large velocity, $f = 0$ as there are no stars with infinitely high velocities. Therefore, surface integration will become zero. Now we get,

$$\frac{\partial \mu}{\partial t} + \frac{\partial (\mu \bar{v}_i)}{\partial x_i} = 0, \tag{2.39}$$

where

$$\mu(\vec{x},t) = \int f(\vec{x},\vec{v},t) d^3\vec{v}, \tag{2.40a}$$

$$\bar{v}_i(\vec{x},t) = \frac{1}{\mu(\vec{x},t)} \int f(\vec{x},\vec{v},t) v_i d^3\vec{v}. \tag{2.40b}$$

Now, multiply equation (2.37) by v_j and integrate again over the whole velocity space,

$$\int v_j \frac{\partial f}{\partial t} d^3\vec{v} + \int v_i v_j \frac{\partial f}{\partial x_i} d^3\vec{v} - \frac{\partial \Phi}{\partial x_i} \int v_j \frac{\partial f}{\partial v_i} d^3\vec{v} = 0,$$
$$\frac{\partial (\mu \bar{v}_j)}{\partial t} + \frac{\partial (\mu \overline{v_i v_j})}{\partial x_i} - \frac{\partial \Phi}{\partial x_i} \int v_j \frac{\partial f}{\partial v_i} d^3\vec{v} = 0 \tag{2.41}$$

The last term of this equation can be simplified using integration by parts and the divergence theorem,

$$\frac{\partial \Phi}{\partial x_i} \int v_j \frac{\partial f}{\partial v_i} d^3\vec{v} = -\frac{\partial \Phi}{\partial x_i} \int \frac{\partial v_j}{\partial v_i} f d^3\vec{v}$$

---

[1] Suppose a scalar product $\vec{A} \cdot \vec{B} = \Sigma_i A_i B_i$, where i = 1, 2, 3 represents x-, y-, z-components, respectively. In summation convention, we can simply write it as $\vec{A} \cdot \vec{B} = A_i B_i$. It automatically assumes summation over dummy index i.

$$
\begin{aligned}
&= -\frac{\partial \Phi}{\partial x_i} \int \delta_{ij} f d^3 \vec{v} \\
&= -\delta_{ij} \frac{\partial \Phi}{\partial x_i} \mu \\
&= -\mu \frac{\partial \Phi}{\partial x_j}.
\end{aligned}
\tag{2.42}
$$

Substituting it in equation (2.41) gives,

$$
\frac{\partial (\mu \bar{v}_j)}{\partial t} + \frac{\partial (\mu \overline{v_i v_j})}{\partial x_i} + \mu \frac{\partial \Phi}{\partial x_j} = 0,
\tag{2.43}
$$

$$
\mu \frac{\partial \bar{v}_j}{\partial t} + \bar{v}_j \frac{\partial \mu}{\partial t} + \frac{\partial (\mu \overline{v_i v_j})}{\partial x_i} + \mu \frac{\partial \Phi}{\partial x_j} = 0.
\tag{2.44}
$$

Making use of equation (2.39) in equation (2.44) returns,

$$
\mu \frac{\partial \bar{v}_j}{\partial t} - \bar{v}_j \frac{\partial (\mu \bar{v}_i)}{\partial x_i} + \frac{\partial (\mu \overline{v_i v_j})}{\partial x_i} + \mu \frac{\partial \Phi}{\partial x_j} = 0.
\tag{2.45}
$$

Now, let us define the velocity dispersion tensor as,

$$
\begin{aligned}
\sigma_{ij}^2(\vec{x}, t) &= \frac{1}{\mu(\vec{x}, t)} \int (v_i - \bar{v}_i)(v_j - \bar{v}_j) f(\vec{x}, \vec{v}, t) d^3 \vec{v} \\
&= \overline{v_i v_j} - \bar{v}_i \bar{v}_j.
\end{aligned}
\tag{2.46}
$$

Substituting $\overline{v_i v_j}$ from equation (2.46) in equation (2.45),

$$
\begin{aligned}
\mu \frac{\partial \bar{v}_j}{\partial t} + \mu \bar{v}_i \frac{\partial \bar{v}_j}{\partial x_i} + \frac{\partial (\mu \sigma_{ij}^2)}{\partial x_i} + \mu \frac{\partial \Phi}{\partial x_j} &= 0, \\
\mu \frac{\partial \bar{v}_j}{\partial t} + \mu \bar{v}_i \frac{\partial \bar{v}_j}{\partial x_i} &= -\mu \frac{\partial \Phi}{\partial x_j} - \frac{\partial (\mu \sigma_{ij}^2)}{\partial x_i}.
\end{aligned}
\tag{2.47}
$$

This equation is similar to Euler's equation in fluid dynamics. Equations (2.39) and (2.47) is known as Jeans equations for stellar dynamics.

2.7   Numerical simulations

Although the analytical description of galactic dynamics is very useful and valuable, it cannot provide us complete information about galaxy formation and evolution. Therefore, numerical simulations come into picture as indispensable tools to study galaxies, thanks to the revolution-

ary development of computers. In the following subsections, we discuss various methods used in numerical simulations.

2.7.1   Force calculation

- **Particle-Particle (PP) algorithm:** Consider a stellar system having N number of identical particles[2] each with mass m. Gravitational force per unit mass on ith particle in the system is simply given by,

$$\vec{F}_i = \sum_{j \neq i}^{N} Gm \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^3}. \tag{2.48}$$

At any time step of the simulation, a computer will perform an order of N^2 calculations to estimate forces on all the particles. This method is fine as long as the number of particles is less than 10^4. For galactic scale simulations, it is computationally a very expensive method.

- **Tree algorithm:** Here, we review Tree algorithm discussed in Barnes & Hut (1986). It reduces force calculations to order of $N \ln N$. In Tree algorithm, the whole simulation is placed inside a cubical box. Next, we divide this box into eight equal cubical sub-boxes. If any sub-box has more than one particle, we divide it again into eight equal cubical sub-sub-boxes until each box has at most one particle. This hierarchical structure is call oct-tree (or quad-tree in 2D). The original cube is the root, the cubes having more than one particle are the nodes, and cubes with at most one particle are leaves.

For force calculation on any subject particle, we walk the tree starting from root nodes until angles subtended by the nodes are less than the tolerance angle (θ_{tol})[3]. Suppose a node of size l is located at a distance r from the subject particle, the condition to stop walking the tree further is given by,

$$\frac{l}{r} < \theta_{tol}. \tag{2.49}$$

At this point, we can treat particles in the node as a single particle of node mass and located at node's center of mass. In Fig. 2.4, we have illustrated the Tree algorithm in two dimensions. The left-hand panel of the figure shows the quad-tree for a given distribution of particles. The particle on which we want to compute force is colored green. The

---

[2] A system with N number of particles is commonly knows as $N-$body system.
[3] Tolerance angle also known as opening angle because it decides whether node is open to walk further or not.

right-hand panel of the figure shows the corresponding closed nodes and their equivalent particles in red color for an arbitrary θ_{tol}. The connecting lines show the forces acting after the implementation of the Tree algorithm.

Fig. 2.4 An illustration of the Tree algorithm for force computation in 2D. The left-hand panel shows the quad-tree for a given distribution of particles. The particle on which we want to compute force is colored green. The right-hand panel shows the corresponding closed nodes and their equivalent particles in red color for an arbitrary θ_{tol}. The connecting lines show forces acting after Tree algorithm is applied.

- **Particle-Mesh (PM) algorithm:** In the Particle-Mesh algorithm, we first divide the whole simulation box of size L into a grid of M^3 meshes with fixed size $l = L/M$. Second, we calculate the mass density at mesh-points. Third, we determine the potential on the mesh-points by solving Poisson's equation. Finally, the gradient of the potential at the mesh-points provides the force on the mesh-points (Hockney & Eastwood, 1988).

Consider an $N-$body system with identical particles each with mass m. The mass density on a mesh-point $\vec{x}_p$ due to particles distribution ($\vec{x}_i$, $i = 1, 2, 3, ..., N$) can be expressed as follows,

$$\rho(\vec{x}_p) = \frac{m}{L^3} \sum_{i=1}^{N} W(\vec{x} - \vec{x}_p). \tag{2.50}$$

Where W is a normalized smoothing kernel. It is chosen in such a way that the total angular momentum of the system is conserved. For example,

$$W(\vec{x}) = \frac{1}{\pi l^3} \begin{cases} \frac{3}{4} - R^2 & (0 \leq R \leq \frac{1}{2}) \\ \frac{1}{2}(\frac{3}{2} - R)^2 & (\frac{1}{2} < R \leq \frac{3}{2}) \\ 0 & \text{otherwise,} \end{cases} \tag{2.51}$$

where $R = |\vec{x}|/l$. Potential $\Phi(\vec{x})$ on a mesh-point can be obtained using Fourier transformation of Poisson's equation as follows,

$$-k^2 \Phi(\vec{k}) = 4\pi G \rho(\vec{k}). \tag{2.52}$$

Now, gravitational force per unit mass at the mesh-point can be calculated using the inverse Fourier transformation of

$$\vec{F}(\vec{k}) = -i\Phi(\vec{k})\vec{k}. \tag{2.53}$$

Finally, the forces on the locations of particles can be obtained using interpolation of forces on mesh-points as follows,

$$\vec{F}(\vec{x}) = \sum_{i=1}^{N} W(\vec{x} - \vec{x}_p)\vec{F}(\vec{x}_p). \tag{2.54}$$

- **P^{3}M algorithm:** P^{3}M is a hybrid algorithm that uses Particle-Particle (PP) algorithm and Particle-Mesh (PM) algorithm simultaneously. In P^{3}M algorithm, Long-range forces are computed using the PM algorithm, whereas the short-range forces are calculated using the PP algorithm. The PP algorithm is usually implemented in the regions where particles are highly clustered. If there are many particles within $2l$ distance, the PP algorithm will be initiated otherwise the PM algorithm is used.

- **TreePM algorithm:** TreePM is also a hybrid algorithm that uses Tree algorithm and Particle-Mesh (PM) algorithm simultaneously. In TreePM algorithm, long-range forces are computed using PM algorithm, whereas short-range forces are calculated using the Tree algorithm.

2.7.2   Gravitational softening

Sometimes, during the evolution of simulation, particles come so close that the force calculated using equation (2.48) diverges. In a collisionless system, such encounters are totally unphysical. To avoid such collisions in the simulation, we soften the gravitational force as follows,

$$\vec{F}_i = \sum_{j \neq i}^{N} Gm \frac{\vec{x}_j - \vec{x}_i}{(|\vec{x}_j - \vec{x}_i|^2 + \varepsilon^2)^{3/2}}, \tag{2.55}$$

where ε is a constant known as gravitational softening length or Plummer softening length. Introducing constant ε in the denominator of force equation does not affect force calculation when $|\vec{x}_j - \vec{x}_i| > \varepsilon$. However, it avoids numerical artifacts when two particles are very close. In such cases, the denominator becomes ε.

2.7.3   Smoothed-particle hydrodynamics (SPH)

Smoothed-particle hydrodynamics (SPH) is one of the well-known techniques for studying gas dynamics in numerical simulations. It is based on the Lagrangian formulation of fluid dynamics where a fluid is treated as a set of N particles. In SPH, we follow the motion of individual particle whose mass m is constant during the evolution (Monaghan, 1992).

To obtain the smoothed form of any fluid field $A(\vec{x})$, such as density (ρ) or temperature (T), in SPH simulations, we convolve the field with some smoothing kernel W as follows,

$$A_s(\vec{x}) = \sum_{i=1}^{N} \frac{m_i}{\rho_i} A(\vec{x}_i) W(\vec{x} - \vec{x}_i, h), \tag{2.56}$$

where h is the characteristic radius of the smoothing kernel. It defines the extent of the kernel, and is typically chosen in such a way that W becomes zero at a distance ηh, where η is generally an order of unity. A commonly used smoothing kernel in SPH is given by,

$$W(\vec{x}, h) = \frac{1}{\pi h^3} \begin{cases} 1 - \frac{3R}{2} + \frac{3R^3}{4} & (0 \leq R \leq 1) \\ \frac{1}{4}(2 - R)^3 & (1 < R \leq 2) \\ 0 & \text{otherwise}, \end{cases} \tag{2.57}$$

where $R = |\vec{x}|/h$ and η is 2.

2.7.4   Time integration

Time integration is a crucial part of any numerical simulation. Various simulations adopt the leapfrog integration scheme to predict the next co-ordinates of a particle based on its initial co-ordinates and for a given time-step. Let us say $\vec{x}(t_n)$, and $\vec{v}(t_n)$ are respectively the position and velocity co-ordinates of a particle at any time t_n. If $\vec{F}(t_n)$ is the force per unit mass acting on the particle at time t_n, the next position and velocity of this particle after Δt_n time-step are calculated using following steps,

$$\vec{x}(t_{n+1/2}) = \vec{x}(t_n) + \frac{\Delta t_n}{2}\vec{v}(t_n), \tag{2.58a}$$

$$\vec{v}(t_{n+1/2}) = \vec{v}(t_n) + \frac{\Delta t_n}{2}\vec{F}(t_n). \tag{2.58b}$$

These are the position and velocity co-ordinates at half time-step. We can calculate force at the intermediate position as follows,

$$\vec{F}(t_{n+1/2}) = -\nabla\Phi|_{\vec{x}(t_{n+1/2})}. \tag{2.59}$$

Now the position and velocity co-ordinates at next half time-step are

$$\vec{x}(t_{n+1}) = \vec{x}(t_{n+1/2}) + \frac{\Delta t_n}{2}\vec{v}(t_{n+1/2}), \tag{2.60a}$$

$$\vec{v}(t_{n+1}) = \vec{v}(t_{n+1/2}) + \frac{\Delta t_n}{2}\vec{F}(t_{n+1/2}). \tag{2.60b}$$

These are the co-ordinate of the particle after Δt_n time-step and new simulation time becomes

$$t_{n+1} = t_n + \Delta t_n. \tag{2.61}$$

2.8   Existing simulations

Several groups of researcher across the globe have performed cosmological simulations that lead to structure formation and galaxy evolution. These simulations have specific science goals. Some simulations adopt large box sizes to understand large scale structure formation and clustering. On the other hand, some simulations focus on small scale clustering and galaxy formation using small box sizes. These simulation can be simply dark matter only or can include sophisticated baryonic physics. On the basis of these factors, simulations can be categorized into four classes:

- **Dark matter only large volume simulations:** These simulations consider all the matter as dark matter and evolve it in a large cosmological box. A typical purpose of these simulations is to investigate the structure formation and gravitational clustering of matter at large scales. They generally have course mass resolution. Bolshoi simulations, Dark Sky, Millennium, Millennium-II, Millennium-XXL are examples of dark matter only large volume simulations.

- **Dark matter only zoom-in simulations:** These simulations are refined versions of dark matter only large volume simulations in the sense that they evolve a small region of interest in large volume simulations at much higher mass resolution. They are useful to study small scale clustering and dark matter halo formation in cosmological environment. Via Lactea, ELVIS, Phoenix, GHALO, Aquarius are the examples of dark matter only zoom-in simulations.

- **Dark matter + baryon large volume simulations:** These simulations treat dark matter and baryonic matter as separate entities. They also include baryonic physics to probe its impact on large scale structure and clustering of baryons along dark matter. EAGLE, Romulus25, Horizon-AGN, Massiveblack-II, Simba, Magneticum, Illustris, IllustrisTNG are examples of (dark matter + baryon) large volume simulations.

- **Dark matter + baryon zoom-in simulations:** Similar to the dark matter only zoom-in simulations, these simulations also evolve a small region of interest from the large volume simulation at much better resolution and include sophisticated baryonic physics. These simulations help us understand galaxy formation, the effect of baryonic physics, dark matter halo abundance, and several other small scale properties in the cosmological scenario. Eris, Latte/FIRE, APOSTLE, NIHAO, Auriga are the examples of dark matter + baryon zoom-in simulations.

2.9   Codes, tools, and data used

Every simulation requires initial realization of the system we are interested in. Next, this realization is evolved in time to understand the dynamics of the system over time. Here, we list the simulation codes, analysis tools, and data used in this thesis work.

- **GALIC:** The initial realizations of the model galaxies are generated using a publicly distributed code GALIC (Yurin & Springel, 2014). GALIC uses predefined analytic density distribution functions to populate the particles and searches for the stable solution of the collisionless Boltzmann Equation (CBE). For a stable solution, it randomly changes

velocities of the particles keeping their positions fixed in space. Some components of the Schwarzschild's method and made-to-measure technique are employed in GALIC code.

- **GADGET-2:** All the initial realizations were evolved in time using a publicly distributed code GADGET-2 (Springel et al., 2001a; Springel, 2005a,b). GADGET-2 is a massively parallelized and adaptive TreeSPH code. It uses $N-$body approach to treat collisionless dynamics and SPH technique to describe hydrodynamics. It makes use of hybrid TreePM algorithm to compute gravitational forces during evolution. The time integration in GADGET-2 is performed using leapfrog method with adaptive time-steps.

- **GALFIT:** For the photometric analysis of simulated galaxies, we used the latest version of the GALFIT code which is a two-dimensional multi-component decomposition tool (Peng, 2003; Peng et al., 2011). It uses pre-defined surface brightness profiles and fits them using the Levenberg–Marquardt algorithm. The best fit solution is determined using chi-square minimization. It is an efficient and powerful tool to perform multi-component fitting simultaneously.

- **IllustrisTNG50:** We have also used the simulated data product from the state-of-art gravo-magnetohydrodynamical simulation, IllustrisTNG50. It is one of the highest resolution runs of IllustrisTNG simulations, a successor of the Illustris project. IllustrisTNG comprises three cosmological volumes, namely TNG50, TNG100, and TNG300 having box sizes approximately 50, 100, and 300 Mpc respectively. All the TNG simulations are performed with the moving-mesh and massively parallel code AREPO (Springel, 2010) starting from the redshit z=127 to z=0. Now, all three runs are publicly available in their entirety (Nelson et al., 2019a).

"Your hand touching mine. This is how galaxies collide."

– **Sanober Khan**

3

Galaxy Flybys: Evolution of the Bulge, Disk, and Spiral Arms

3.1   Introduction

There are basically two types of galaxy interactions; mergers and flybys. It is well known that both phenomena play a major role in the hierarchical growth of the large scale structure (Aarseth et al., 1979; Frenk et al., 1985), as well as produce morphological changes in galaxies (Martin et al., 2018). In the case of flybys, although the galaxies finally separate without merging, the interaction can have a strong effect on the galaxy disks (Arp, 1966). Flybys can be further categorized into major and minor interactions, depending upon the primary (host) to secondary (satellite) galaxy mass ratio. In general, if the ratio is greater than 3, it is classified as a minor interaction otherwise it is called a major interaction (Stewart, 2009). At early epochs, the universe was smaller in size and mergers were more frequent, hence they played an important role in the growth of galaxies (Kauffmann et al., 1993a; van den Bergh et al., 1996; Murali et al., 2002; Stewart et al., 2008). However, recent studies have shown that at later times, as the universe expanded, flybys became more common than mergers (Sinha & Holley-Bockelmann, 2012a). Thus, although mergers may appear more important for changing

galaxy morphology, the cumulative effect of flyby interactions may also be important since they are more frequent than mergers in the cosmic history of galaxies (An et al., 2019a).

Early simulations clearly show the importance of flybys on the evolution of galaxies (Toomre & Toomre, 1972; Barnes & Hernquist, 1992). Later studies have found that flybys play a crucial role in producing tidally induced bars (Berentzen et al., 2004; Lang et al., 2014; Łokas et al., 2014). The associated mass inflow and bar evolution can result in the formation of pseudobulges (Weinzirl et al., 2009a) as well as kinematically decoupled cores (De Rijcke et al., 2004). The tidal interaction of flybys have an even stronger effect on the disks of the major galaxy, inducing the formation of spiral arms (Dubinski et al., 1999; Renaud et al., 2015; Moreno et al., 2019) that result in strong star formation (Duc et al., 2018a). If the gas infall towards the center is large enough, then these effects can ultimately lead to starbursts (Mihos & Hernquist, 1994a) and the triggering of active galactic nuclear (AGN) activity in galaxies (Combes, 2001a). Studies have shown that the tidal interaction can also result in the formation of warps in galaxy disks (Kim et al., 2014a), as well as increase the stellar velocity dispersion, resulting in disk thickening (Reshetnikov & Combes, 1997a). Other effects of flybys on stellar disks are the formation of stellar streams and tidal bridges (Duc & Renaud, 2013a). Studies suggest that flybys will also affect galaxy spin, although the effect is dependent on several factors such as orbit direction and the nature of the galaxies (Choi & Yi, 2017a).

In this study we investigate the effect of flybys on the two important stellar components of galaxies, their bulges and their disks. Bulges can be broadly classified into two types based on their morphology and the dynamics of their constituent stars; classical bulges that are approximately spherical in shape, and pseudobulges that appear disky and flatter in shape (Kormendy et al., 2006a; Drory & Fisher, 2007a; Athanassoula, 2008; Fisher & Drory, 2008a,d). Classical bulges are dynamically hotter systems compared to pseudobulges that have more disky orbits (Kormendy, 1993a; Andredakis & Sanders, 1994a). Classical bulges are thought to form from the monolithic collapse of gas clouds or clumps at early epochs, and then grow from the accretion of smaller galaxies (Aguerri et al., 2001a; Bournaud et al., 2005a; Brooks & Christensen, 2016).

Pseudobulges can be further divided into two categories: disky pseudobulges and boxy/peanut pseudobulges. Disky pseudobulges are circular in shape but in the vertical direction they are as flat as the disks of their host galaxy. Hence, it is nearly impossible to detect them in edge-on galaxies. They are thought to form within the inner disks via star formation (Laurikainen et al., 2009). Boxy/peanut bulges are more extended in the vertical direction and can hence be detected in edge-on galaxies. They are usually associated with bars in disk galaxies (Friedli & Pfenniger, 1990; Debattista et al., 2006; Gadotti, 2011). Boxy/peanut pseudobulges, unlike classical bulges, are formed due to the secular evolution of galaxy disks, the buckling instability

of bars (Combes & Sanders, 1981), or via mergers with gas rich galaxies (Keselman & Nusser, 2012). Using observations of galaxies in our local universe at distances less than 11 Mpc, Fisher & Drory (2011a) found that 80% of the galaxies with a stellar mass more than $10^9 M_\odot$ contain either a pseudobulge or no bulge. This domination of pseudobulges in the local universe challenges the hierarchical models of galaxy formation, according to which the majority of galaxies should have classical bulges (Frenk et al., 1985).

In the literature, there have been several observational and numerical studies on the formation and evolution of bulges (Kormendy & Kennicutt, 2004a; Athanassoula, 2005a; Gadotti, 2009a; Laurikainen & Salo, 2016a). It is clear that both secular processes and violent ones such as galaxy mergers are important for galaxy evolution (Tonini et al., 2016). But most of the literature is biased towards only one kind of galaxy interaction – galaxy mergers. The other kind of galaxy interaction, flybys, are largely ignored. This could be due to two reasons. The first is that the effect of flybys is lower than that of mergers. However, recent cosmological simulations have shown that galaxy flybys are as frequent as mergers in the low redshift universe (Yee & Ellingson, 1995; Sinha & Holley-Bockelmann, 2012a) and they may play an equally prominent role as mergers in the evolution of galaxies (Di Matteo et al., 2007; Li et al., 2008; Kim et al., 2014a; Sinha & Holley-Bockelmann, 2015a). The second reason is that bulges lie deep within the potential of a galaxy and the more massive ones make disks extremely stable against bar instability (Kataria & Das, 2018; Kataria et al., 2020). The spherical non-rotating classical bulge cannot be torqued effectively in flyby interactions unless there is some instability e.g. bar (Saha et al., 2012a, 2016a). Bars can be torqued in flyby interactions (Łokas et al., 2014) and so boxy/peanut pseudobulges can also be torqued because they are essentially a part of the bar. The tidal torque of the satellite galaxies may thus change the kinematics of bars and their associated pseudobulges.

The tidal forces on galaxy disks due to flybys, are known to result in extended spiral arms. They can be similar to grand design spirals such as M51 (Dobbs et al., 2010), or result in extremely large spiral arms such as in NGC 6872 (Horellou & Koribalski, 2007). The interaction can lead to the formation of tidal tails, bridges and warped disks, but the overall morphology depends on many factors such as the pericenter distances, the galaxy orbits, and the relative masses of the galaxies (Oh et al., 2008). Flybys may also affect the spin of galaxy disks, especially when the integrated effect of several interactions is included (Lee et al., 2018). However, although the effect of tidal forces on galaxy disks has been explored in several earlier numerical studies (Walker et al., 1996), it has not been quantified in terms of disk perturbation parameters, such as the Fourier components A_2/A_0, which is useful for measuring the effect of the tidal interaction, and is widely adopted in studies of bars (Kataria & Das, 2019).

Thus, although there are several numerical studies of the tidal effect of flybys on galaxy disks, not much attention has been paid to its effect on the bulges or on quantifying the effects. For example, examining whether there are any changes in bulge Sérsic index or the strength of the spiral arms. In this chapter we try to measure the changes in bulge and disk properties for different pericenter distances. We also include both classical and boxy pseudobulges in our models, disky pseudobulges will be studied later as their evolution possibly involves star formation. Although galaxy interactions do have a large parameter space, for simplicity in this study we focus on one of the parameters that causes the largest change in galaxy morphology – the pericenter distance.

In this study we have used a simple N-body approach; the effect of gas will be included in a future study. In the following sections we first discuss our simulation method which includes creating galaxy disks with classical and pseudobulges, describe how we have analysed the bulges and spiral arms, present our results and then discuss the implications of our work for observational studies.

3.2   Simulations And Analysis

In our simulations, we have fixed the mass ratio of the interacting galaxies at 5:1 and 10:1, as these ratio are similar to a wide range of minor flyby interactions in realistic scenarios. We have also restricted the disk angular momentum vectors to be perpendicular to the orbital plane of the interacting galaxies. The choice of parallel directions of the disk angular momenta of the major and minor galaxies was done to ensure maximum resonance (quasi-resonance) between the angular velocities of stars in the major galaxy and the peak angular velocity of the minor galaxy (D'Onghia et al., 2010).

3.2.1   Galaxy Model: Halo and Disk

We generated our model disk galaxies using the publicly available code GalIC (Yurin & Springel, 2014). Each galaxy has a live dark matter halo, and a stellar disk. In our models, the dark matter halo has a spherically symmetric Hernquist density distribution (Hernquist, 1990),

$$\rho_{dm}(r) = \frac{M_{dm}}{2\pi} \frac{a}{r(r+a)^3} \tag{3.1}$$

where 'a' is the scale radius of the dark matter halo and is related to the concentration parameter 'c' of a corresponding NFW halo (Navarro et al., 1996) of mass $M_{dm} = M_{200}$ in the following

manner,

$$a = \frac{r_{200}}{c} \sqrt{2 \left[\ln\left(1+c\right) - \frac{c}{(1+c)} \right]} \tag{3.2}$$

where r_{200} is the virial radius of the galaxy (which is defined as the radius within which the mean density is 200 times the critical density of the universe) and M_{200} is mass within the virial radius.

The stellar disk density has an exponential form in the radial direction and a sech^2 form in the vertical direction

$$\rho_d(R,z) = \frac{M_d}{4\pi z_0 R_s^2} \exp\left(-\frac{R}{R_s}\right) \text{sech}^2\left(\frac{z}{z_0}\right) \tag{3.3}$$

where M_d is the total mass of the disk, z_0 is the scale height of the disk and R_s is the scale radius of the disk.

3.2.2   Galaxy Model: Bulge

To understand the effect of flyby events on the bulges and disks of galaxies, we have generated two types of bulges in the disk of the larger galaxy, which we call the major galaxy. The first is the classical bulge. Its density is derived from a spherically symmetric Hernquist potential (Hernquist, 1990),

$$\rho_b(r) = \frac{M_b}{2\pi} \frac{b}{r(r+b)^3} \tag{3.4}$$

where M_b is the total mass of the bulge and 'b' is the scale radius of the bulge.

To obtain a pseudobulge in the disk of the major galaxy, we generated it naturally through the bar formation in a bulgeless galaxy. At a fixed disk scale height to scale radius ratio, we varied the angular momentum fraction of the disk to halo so that it can form a bar which can spontaneously buckle to form a pseudobulge (boxy/peanut bulge) after evolution. We evolved this model upto 10 Gyrs and calculated the bar strength using the ratio of m=2 to m=0 Fourier modes. The amplitudes of the mth Fourier mode at a radius R is given by,

$$a_m(R) = \sum_{i=1}^{N} m_i(R) \cos(m\phi_i), m = 0, 1, 2, 3, ... \tag{3.5}$$

$$b_m(R) = \sum_{i=1}^{N} m_i(R) \sin(m\phi_i), m = 0, 1, 2, 3, ... \tag{3.6}$$

Fig. 3.1 The evolution of bar strength in the pseudobulge model until 10 Gyrs. Due to the buckling of the bar (the 2 peaks in the $\frac{A_2}{A_0}$ value), the bar strength changes in the initial stage of evolution. After 6 Gyrs, it shows a nearly constant strength. This is where we take our initial model for a galaxy with a pseudobulge.

where $m_i(R)$ and ϕ_i are the mass and the azimuthal angle of ith particle at radius R respectively, and N is the total number of particles at the same radial position. The strength of a bar is defined as

$$\frac{A_2}{A_0} = max\left[\frac{\sqrt{[a_2(R)]^2 + [b_2(R)]^2}}{\sum_{i=1}^{N} m_i(R)}\right] \tag{3.7}$$

Fig. 5.2 shows the evolution of bar strength for the major galaxy with a pseudobulge (PG_{PB} model) upto 10 Gyrs. The fluctuations in the bar strength at early epochs of the evolution is the indicator of bar buckling. The bar strength remains approximately constant after 6 Gyrs of evolution. This is where we take the initial conditions for our pseudobulge model i.e. we took the galaxy at 6 Gyr as the initial model for simulations of galaxy flybys with pseudobulges. The formation of the pseudobulge is graphically represented in fig. 3.2 as a time sequence of the galaxy evolution, using the edge-on view of the disk and bar.

3.2.3   Choice of Model Parameters for Simulations

Table 5.1 summarises the initial parameters of the primary (major) and secondary (minor) galaxies. The mass of the major galaxy is similar to that of the Milky Way galaxy in all the

Fig. 3.2 The visual illustration of the pseudobulge formation. The top to bottom panels shows the time sequence of the edge-on view of the major galaxy, perpendicular to the bar, together with similar isodensity contours. From top to bottom, the five panels show the snapshots at t=0.0, 0.3 (1st peak in the bar strength), 1.7 (2nd peak in the bar strength), 3.0 (after the buckling), and 6.0 Gyr (beginning of the constant bar strength) respectively. The bottom panel displays the initial conditions for the flyby models of galaxies with a pseudobulge .

Table 3.1 Initial parameters of the primary (major) and secondary (minor) galaxies. In this table, PG_{CB} = Primary galaxy with classical bulge, PG_{PB} = Primary galaxy with pseudobulge, SG_5 = Secondary galaxy with 1/5 mass of primary galaxy, and SG_{10} = Secondary galaxy with 1/10 mass of primary galaxy. (units $M_\odot$ = solar mass, kpc = kiloparsec).

Parameters	PG_{CB}	PG_{PB}	SG_5	SG_{10}
Total mass (M)	$1.2 \times 10^{12} M_\odot$	$1.2 \times 10^{12} M_\odot$	$2.4 \times 10^{11} M_\odot$	$1.2 \times 10^{11} M_\odot$
Halo spin parameter(λ)	0.035	0.035	0.035	0.035
Halo concentration (c)	10	10	11	11
Disk mass fraction	0.025	0.03	0.01	0.01
Bulge mass fraction	0.005	0.0 [a]	0.002	0.002
Disk spin fraction	0.03	0.022	0.01	0.01
Disk scale radius (R_s)	3.80 kpc	2.30 kpc	1.95 kpc	1.55 kpc
Disk scale height (z_0)	0.38 kpc	0.23 kpc	0.195 kpc	0.155 kpc
Halo particles(N_{Halo})	2.5×10^6	2.5×10^6	5.0×10^5	2.5×10^5
Disk particles(N_{Disk})	1.5×10^6	2.5×10^6	3.0×10^5	1.5×10^5
Bulge particles(N_{Bulge})	1.0×10^6	0 [a]	2.0×10^5	1.0×10^5
Total particles(N_{Total})	5.0×10^6	5.0×10^6	1.0×10^6	5.0×10^5

[a] Initially this model does not have any bulge particles. A pseudobulge grows from the disk itself after a bar forms and buckles.

models, as suggested by many studies (see Wang et al. (2020a) and reference therein). The halo spin parameter (λ) of the galaxies is chosen to be 0.035 which corresponds to the peak of the halo spin distribution given in Bullock et al. (2001). The values of concentration parameter (c) lie well within the mass-concentration relation given by Wang et al. (2020b). The stellar mass fraction of the galaxies are chosen from the stellar to halo mass relation (SHMR) (Behroozi et al., 2013; Moster et al., 2013). The choice of disk scale length for the major galaxy is made such that the model PG_{CB} does not show any bar feature after evolution but the model PG_{PB} develops a bar which then buckles to form a pseudobulge.

To generate a bar stable major galaxy (hereafter referred to as an unbarred galaxy) and bar unstable major galaxy (hereafter referred to as a pseudobulge galaxy), we tuned the angular momentum fraction of the disk to halo, at a fixed disk scale height to scale radius ratio. This tuning of angular momentum decides the size of the disk (see Mo et al. (1998) for the galaxy models). The unbarred galaxy models did not show any signatures of bar formation throughout the simulation time of 5 Gyrs but the barred galaxy forms the bar and buckles to form a pseudobulge. In fig. 3.3, we have shown the rotation curves (left panel) and surface densities (right panel) of the major galaxy models.

The choice of the number of particles in the major galaxy is based on the region of interest within the galaxies. Since we are interested in the bulge and the disk of the major galaxy, we selected the number of particles in the major galaxy in such a way that the two body relaxation

Fig. 3.3 The left panel shows the total rotation curves of initial models of the major galaxy. The panel on the right shows the corresponding stellar surface densities of the galaxy. The red dotted curve and the blue dashed curve represent the PG_{CB} and PG_{PB} models respectively.

time is greater than the time of interest (i.e. the simulation time) in the central region of the galaxy (Power et al., 2003). The relaxation time is 5 Gyr for the models PG_{CB} and PG_{PB} at the radii 0.14 kpc and 0.27 kpc respectively. To confirm that our results do not depend on mass resolution of the simulations, we have also performed four high mass resolution simulation runs (one isolated and one $r_p = 40\ kpc$ for both PG_{CB} and PG_{CB} models) with twice as many particles as listed in the Table 1.

After generating the major and minor galaxies, we put them on hyperbolic orbits of eccentricity $e = 1.1$ as illustrated in fig. 3.4 so that the minor galaxy does not decay by dynamical friction of the major galaxy. In this figure, the minor galaxy is shown at time intervals of 0.5 Gyr and the major galaxy is shown at time t=2.0 Gyr. The solid red curves traces the actual paths of the center of mass of the two galaxies in one of our simulations. The initial separation of the galaxies was chosen to be 255 kpc which is the sum of the virial radii (r_{200}) of the major and minor galaxies. We evolved the galaxies for 4 Gyr using the publicly available massively parallel code Gadget-2 (Springel et al., 2001a; Springel, 2005a,b). The evolution time is set to 4.0 Gyr because galaxies are well separated after evolving for this much amount of time. In all of our simulations, the gravitational softening is 0.02 kpc for the stellar particles and 0.03 kpc for the halo particles. The change in total angular momentum is within 0.15% for all the simulations in 4 Gyrs of evolution, which ensures an output with minimum numerical errors.

To study the effect of flyby interactions on the bulge and the disk of the major galaxy, we varied the distance of closest approach i.e. the pericenter distance of the galaxies (r_p) from

Fig. 3.4 The orbits of two disk galaxies of mass ratio 10:1 undergoing a flyby interaction with pericenter distance of 40 kpc. The red solid curves trace the orbits of the galaxies during flyby. The minor galaxy is shown in time steps of 0.5 Gyr and the major galaxy is shown at the time 2 Gyr. The mass weighted particle number density is represented by the color on the log scale. In this illustration, only the stellar particles are shown.

40 kpc to 80 kpc, keeping all the other parameters fixed. Hence, we have two sets of simulation models for each pericenter distance (r_p), which are : (1) a major galaxy with a classical bulge and (2) a major galaxy with a pseudobulge bulge. Each model is further simulated for 10:1 and 5:1 galaxy mass ratios. As a control model, we have also evolved the major galaxy in isolation for the same time period of 4 Gyr. The isolated model provides the benefit of removing the effect of secular evolution and the effect of discreteness, if any.

For naming the models, we have used pericenter distances determined assuming 2-body systems. For e.g. $r_p = 40$ denotes the model with pericenter distance = 40 kpc as determined from a two body system. However, the real N-body systems always deviates from these pre-determined pericenter distances. Note that all the quantities are in units of the Hubble parameter 'h' (where the Hubble constant is $H_0 = 100\ h\ km\ s^{-1} Mpc^{-1}$ and $h = 0.67$ from Planck Collaboration et al. (2020)) and all the plots are for the major galaxy unless otherwise specified.

3.2.4   Analysis

We used the latest released version of GALFIT, which is a widely used two dimensional galaxy image decomposition tool, for the bulge-disk decomposition of the simulated galaxies (Peng, 2003; Peng et al., 2011). Since we are interested in the effect of the flyby on the major galaxy

or host galaxy only, we did the two dimensional image decomposition of the major galaxy and not the minor galaxy. Also, since GALFIT requires the galaxy images in 'fits' format as an input, we first made 'fits' format images of the galaxies using simulation snapshots at time steps of 0.2 Gyr. For the galaxy decomposition, we did not consider any background sky and hence the point spread function (psf) is a delta function. We used a simple exponential profile for the two dimensional fitting of a face-on disk,

$$\Sigma_d(R) = \Sigma_{d0} \exp\left(-\frac{R}{R_s}\right) \tag{3.8}$$

where Σ_{d0} is the central surface density of the disk and R_s is the disk scale radius. For the two dimensional fitting of an edge-on disk, we used following profile

$$\Sigma_d(R,z) = \Sigma_{d0}\left(\frac{R}{R_s}\right) K_1\left(\frac{R}{R_s}\right) \operatorname{sech}^2\left(\frac{z}{z_0}\right) \tag{3.9}$$

where K_1 is a Bessel function and z_0 is the scale height of the disk. We used a Sérsic profile for the two dimensional fitting of the bulge

$$\Sigma_b(R) = \Sigma_{b0} \exp\left[-b_n\left\{\left(\frac{R}{R_e}\right)^{\frac{1}{n}} - 1\right\}\right] \tag{3.10}$$

where R_e is effective radius of the bulge, Σ_{b0} is the surface density of the bulge at R_e, and b_n is a function of the Sérsic index n. Previous studies of galaxy bulges suggest that the Sérsic index $n = 2$ is a good proxy for distinguishing between classical bulges and pseudobulges (Fisher, 2006a; Fisher & Drory, 2008d). Generally, classical bulges have a Sérsic index $n > 2$ while pseudobulges have a Sérsic index $n < 2$. We have applied this classification scheme for bulges in our study.

For improving the goodness of fit, we did several trials by setting different box sizes, bin sizes (pixel size), and convolution box sizes. We chose all these parameters in such a way that there is minimum effect of over-fitting and under-fitting. Therefore, we centered the major galaxy in a box of 80 kpc $\times$ 80 kpc size with a pixel size of 0.08 kpc $\times$ 0.08 kpc. For each fitting, the initial guess parameters of the galaxy components were chosen visually. See fig. 3.5 for a visual illustration of the fitting procedure in face-on galaxies. The original, fitted and residual distributions of the simulated galaxies with classical bulges are shown in the first, second and third column respectively. Each row of the figure represents the two dimensional bulge-disk decomposition of a model galaxy at a given time as written in the legend of the first column. The figure suggests that GALFIT fits our simulated galaxies quite well with minimum residuals.

Fig. 3.5 The two dimensional decomposition of the major galaxy with a classical bulge. The left, middle and, right columns show the original images, fitted images, and residual images respectively. The mass weighted particle number density is represented by the color on the log scale. For the sake of visualization, only positive pixels are shown in the residual images. From top to bottom, the five rows show the model galaxy at t=0.0 Gyr, *isolated* model at t=2.0 Gyr, $r_p = 80$ model at t=2.0 Gyr, $r_p = 60$ model at t=2.0 Gyr, and $r_p = 40$ model at t=2.0 Gyr respectively

Fig. 3.6 One dimensional density profiles of pseudobulge galaxy model and fit obtained from its two dimensional decomposition. The black circle-solid curve, green dash-dotted curve, blue dotted curve, and red dashed curve demonstrate the total, disk, pseudobulge, and disk + pseudobulge profiles respectively. The disk profile is smoothly truncated in the inner region because most of the matter is swept by the bar. Our model pseudobulge fits well with $n = 1.4$ Sérsic index.

In our simulations, a pseudobulge is formed from the buckling of the bar. So the bar and pseudobulge are essentially the same component of the galaxy. Fig. 3.6 shows the 1-d density profiles of the pseudobulge galaxy model and the fits obtained from its 2d decomposition using GALFIT. It also includes the individual profiles of the disk and pseudobulge components. The disk profile is smoothly truncated in the inner region of the galaxy because most of the central mass is swept up by the bar. Our model pseudobulge fits well with $n = 1.4$ Sérsic index which verifies its flat nature. However, the morphological decomposition of a galaxy reveals only the projected distribution of mass or light in the different components. This is fine for a spherical bulge but a flat bulge requires more detailed decomposition. Therefore, we explored kinematic methods to decompose the pseudobulges (Abadi et al., 2003).

In the kinematic method, we used the angular momentum of the particles perpendicular to the galaxy plane to separate the 'cold' disk particles and 'hot' bulge particles that had larger velocity dispersion. Let L_z be the angular momentum of a particle perpendicular to the galaxy plane, let E be it's total energy, and $L_c(E)$ be the maximum angular momentum for the particle with energy E in the galaxy potential. Now the ratio $|L_z/L_c(E)|$ will be close to one for cold/disk particles and close to zero for hot/bulge/halo particles (see the fig. 3.7). For the calculation of $L_c(E)$, we divided all the particles in $\sim$200 equal size energy bins and found the particle with maximum L_z in each bin. These L_z are the $L_c(E)$ in the corresponding energy

Fig. 3.7 An example of the kinematic decomposition of the bulge (hot) and disk (cold) components of the galaxy with a pseudobulge (model PG_{PB}). The X-axis denotes the ratio of stars' angular momentum perpendicular to the disk plane (L_z) and the maximum circular angular momentum ($L_c(E)$) corresponding to the stars' energy E. The peak around $\varepsilon = 1.0$ shows the rotation dominated component and the peak away from $\varepsilon = 1.0$ shows the dispersion dominated component. The different colors show the bulge-disk decomposition of the *isolated* model with a pseudobulge (PG_{PB}) at time intervals of 1.0 Gyr.

bins. After that, the $L_c(E)$ of all the particles are calculated using $3rd$ order spline interpolation of these ~ 200 values.

All of our models show a two armed spiral structure during the flyby interactions, which are radially symmetric to each other (see the fig. 3.5). The spiral arms produced in flybys are very close to the logarithmic shape. So we used the Fourier analysis method as discussed in Sellwood & Carlberg (1984a) and Sellwood & Athanassoula (1986) to calculate the strength and pitch angle of the spiral arms. The Fourier coefficients are given by the following expression,

$$A(m,p) = \frac{1}{N} \sum_{j=1}^{N} \exp\left[i\left(m\phi_j + p\ln R_j\right)\right] \qquad (3.11)$$

where m is the number of spiral arms ($m = 2$ for our models), p is a real number related to the pitch angle (α) of the spiral arms, N is the number of stars in a given annular region from radii R_{min} to R_{max}, and (R_j, ϕ_j) are the polar coordinates of the jth star. The range R_{min} to R_{max} is chosen in such a way that the maximum spiral arm strength lies in this range and no

extra feature (e.g. bar and tidal arms) falls within this range. Then we calculated $A(m, p)$ for $p \in [-50, +50]$ at a step $dp = 0.25$ and determine the parameter p_{max} such that the value of $A(m, p)$ maximizes at $p = p_{max}$ (Puerari et al., 2000). So then the pitch angle is given by $\alpha = \arctan(m/p_{max})$ (Oh et al., 2015a; Semczuk et al., 2017) and the spiral arm strength is given by $|A(m, p_{max})|$. For comparison, we also calculated the spiral arm strengths using the method as described for bars in equation 5.5. Both the methods give similar spiral arm strengths.

3.3 Results

We have simulated flyby interactions for galaxies with mass ratio 10:1 and 5:1, and for two types of bulges in the major galaxy: the classical bulge and the pseudobulge. The pericenter distances are set at 40 kpc, 60 kpc, and 80 kpc. As a control case, we have evolved isolated galaxy models for the same length of time. In the following subsections we discuss our results of the effect of galaxy flybys on the disks and bulges. As mentioned earlier, we focus on the properties of the major galaxy only, and not the minor one.

3.3.1 Disk Scale Radius

During close flybys, galaxies experience strong tidal forces due to each others gravity. This tidal force pulls out the stellar mass from the galaxies and results in the formation of strong spiral arms and tidal streams as shown in the third column of fig 3.5. Since the spiral arms form from the stellar particles of the pristine disk, there will be some resultant changes in the disk surface densities. The question is how does the formation of spiral arms affect the pristine disk? To answer this question, we examined the time evolution of the percentage change in the disk scale radius (R_s) of the unbarred, classical bulge galaxies in flybys with 10:1 and 5:1 mass ratios, relative to a control isolated model, as shown in column (a) and column (b) of fig 3.8. The top and bottom rows are the corresponding face-on and edge-on fits of the galaxies. We have performed the edge-on fitting of the galaxies for two perpendicular viewing angles to minimize the bias of the viewing angle on the morphology of the galaxy. The bottom row shows the percentage change in the average disk scale radius for the edge-on fitting. At the beginning of the simulation, the face-on and edge-on fittings give the disk scale radius to be 3.65 kpc and 3.78 kpc respectively. The scale radius from edge-on fitting is nearly equal to the theoretical scale radius (3.8 kpc) but the face-on fitting gives slightly smaller disk scale radius. The cause of this mismatch is the discreteness of the matter distribution in the simulations, which lacks a smooth distribution of particles in the outermost region. Therefore the outermost particles are

Fig. 3.8 The evolution of the percentage change in the disk scale radius of the unbarred, classical bulge galaxy due to flybys. The left and right columns show the flyby simulations of 1/10 and 1/5 galaxy mass ratios respectively. The top and bottom rows represent the outputs from face-on and edge-on galaxy fitting respectively. The red circle-solid curves, the green down triangle-solid curves, the blue upper triangle-solid curves, and the cyan diamond-solid curves represent the models *isolated*, $r_p = 80\, kpc$, $r_p = 60\, kpc$, and $r_p = 40\, kpc$ respectively.

excluded in the face-on fitting but edge-on fitting does not have this drawback. However, this small mismatch is not a problem for studying the disk evolution in our simulations.

From this figure (fig 3.8) it is very clear that the scale radius of the unbarred, classical bulge galaxy decreases with time. The magnitude of the decrease in disk scale radius depends on the pericenter distances of the galaxies. The smaller the pericenter distance, the larger is the change in disk scale radius. Note that the isolated models do not show any change in the scale radius because they are control models and all the change are measured with respect to them. The closest flyby models, which experience the largest tidal forces, show the greatest change in the scale radius because they also have the strongest spiral features. Hence the changes in the disk scale radii are correlated with the formation of the spiral arms (which are discussed later in this section). All the models show a significant change in the disk scale radius just after the pericenter passage, at approximately t=1.0 Gyr. Before pericenter passage, all the models show no significant change in the disk scale radius. In both 10:1 and 5:1 flybys, the face-on fittings show a larger change compared to the edge-on fitting. This is due to the presence of spiral arms which remain in the residual in the face-on fits. However, in the edge-on fits a significant fraction remains in the disk. For a given pericenter distance (r_p), the 5:1 simulations show more changes than the 10:1 simulations. Similarly, for a given major to minor galaxy mass ratio, the closest pericenter passage shows more change than the relatively farther pericenter passage. The effects of close flybys are short lived. As the minor galaxy goes away, the matter pulled out due to the flyby falls back into the major galaxy and starts building the disk again.

3.3.2 Disk Scale Height or Thickness

The minor flyby interactions of galaxies heats up the minor galaxy dynamically and can also induce disk instabilities (e.g. bar formation and disk warping) in the minor galaxy which results in the thickening of the disk (Łokas et al., 2014; Gajda et al., 2018; Łokas, 2018). Although the minor galaxy exerts a much weaker tidal force on the major galaxy, the perturbation may dynamically heat up the major galaxy disk as well. To see if there is any disk thickening for our simulations of flybys with unbarred, classical bulge galaxies, we have plotted the percentage change in the disk scale height (z_0) and the ratio of disk scale height to disk scale radius ($\frac{z_0}{R_s}$) relative to the control isolated model, as a function of time in the top and bottom rows of the fig. 3.9 respectively. The left (a) and right (b) columns of the figure represent the galaxies in 10:1 and 5:1 mass ratio flybys respectively. To reduce the bias of the viewing angle on the morphology of the galaxies, we have plotted average disk scale heights and the ratio of the average disk scale height to average scale radius of two fitted models at perpendicular viewing angles. From the first row of the figure, we can see that both the models show very small or insignificant disk thickening due to the dynamical heating by the minor flybys. The model

Fig. 3.9 Evolution of the percentage change in the disk height or thickening in unbarred galaxies with bulges due to flybys. Left and right columns show the flyby simulations of galaxies with $1:10$ and $1:5$ mass ratio respectively around the major galaxy with classical bulge. Top and bottom rows represent the change in the disk scale height and the ratio of the disk scale height to the disk scale radius respectively. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\ kpc$, $r_p = 60\ kpc$, and $r_p = 40\ kpc$ respectively.

$r_p = 40\,kpc$ for a mass ratio of 5:1 shows a maximum of $\sim 4\%$ increase in the disk scale height due to the flyby. These results indicate that minor flybys cannot heat or thicken the disks of the major galaxy significantly. Only the very close and nearly equal mass flybys (major flyby) can heat the disk of the major galaxy significantly.

However, the disk thickness can also be measured relative to the disk scale radius at the time of measurement as shown in the bottom row of the figure (fig. 3.9). To bring out the effect of flybys on the disk thickening, we have shown the percentage change in the ratio of the disk scale height to the disk scale radius relative to that of control isolated model. At the beginning of the simulation, the disk scale height is $0.1R_s$ which is equal to the theoretical value we started with. As we go forward in time, all the models irrespective of the flyby mass ratio follow the same track until nearly t=1.0 Gyr i.e. before close approach. After that, all the models move on different tracks. In the 10:1 models, the disk thickening remains nearly the same after the passage of the minor galaxy, but the 5:1 models show sudden disk thickening just after the flyby, and then thinning due to increasing disk scale length as discussed in subsection 3.3.1. From both panels of the bottom row, it is clear that the closest approach produces the largest effect on disk thickening at a given major to minor galaxy mass ratio. Similarly, the major flyby produces the greatest effect on disk thickening at a given pericenter passage.

3.3.3 Spiral Arms Strength

The tidal force of the minor galaxy pulls out stellar particles from the major galaxy. At the same time, the resonance between the stellar orbits and the orbit of the minor galaxy helps the major galaxy to develop spiral arms. In our simulations, the galaxies move in a prograde-prograde configuration which is the most favourable orientation for the formation of spiral arms. All of our models develop two spiral arms during the interactions which are radially symmetric to each other (see the fig. 3.5). The strength of spiral arms in a galaxy represents how well the spiral features are distinguishable from the host disk. Hence the amplitude of the m=2 Fourier mode will be a good estimator of the spiral arm strength at any given radius (see equation 4.8).

In fig 3.10, we have plotted the strength of the spiral arms for our models. The left and right columns of the figure show the strength of spiral arms in the flyby models with mass ratios 10:1 and 5:1 respectively. The top and bottom rows represent the spiral arms strength in galaxies with a classical bulge and a pseudobulge respectively. From all the panels of the figure, one can see that the major galaxy in all the flybys shows a sudden increase in the spiral strength at approximately t=1.0 Gyr which is the time of pericenter passage of the galaxies. There is no change or negligible change in the strength of the spiral arms for the control or isolated models throughout the simulation. This shows the importance of flyby interactions in the formation of strong spiral arms.

Fig. 3.10 Time evolution of the spiral arms strength. The left and right columns show the flyby simulations of mass ratios 10:1 and 5:1 respectively. The top and bottom rows represent the major galaxy model of classical bulge and pseudobulge respectively. The red, green, blue, and cyan curves represent the models *isolated*, $r_p = 40\ kpc$, $r_p = 60\ kpc$, and $r_p = 80\ kpc$ respectively.

The time evolution of the spiral arms strength in the classical bulge models shows that the strength increases as the satellite crosses the pericenter, which is at approximately 1.0 Gyr. After some time, the magnitude of the arm strength reaches its maximum and starts decreasing slowly. As expected the value of the peak strength increases with decreasing pericenter distance of the galaxies. Also, at a given pericenter distance, the 5:1 flyby models show higher peak strength than the 10:1 flyby models. The closer flybys attain peak strength earlier than the farther flybys. However, although the closest flyby interactions show the most rapid growth in spiral arms strength, they also show rapid decay in the spiral arms strength. As a result all the flyby models reach the same strength level after some time. This result shows the importance of satellite galaxies or massive clusters as spiral strength boosters. They help the major galaxies to maintain well defined and long lasting spiral arms.

Also, for the close flybys of the unbarred, classical bulge models, the spiral arms can be traced all the way to the center of the galaxy. For example, in the fig. 3.5, the $r_p = 40 \, kpc$ model shows a grand design spiral structure after the flyby. The two spiral arms are distinct all the way to the center. It is well known that the strength of global disk instabilities in disks (e.g. bars, spiral arms) depend on the mass and the concentration of the bulge (Shen & Sellwood, 2004; Athanassoula et al., 2005; Kataria & Das, 2018). We see this effect in our simulations as well, since both the strength and the extent of the spiral arms formed in the flybys depends on the distance of closest approach and the presence of the bulge mass.

The evolution of spiral arms strength in the pseudobulge models is even more interesting as can be seen from the bottom row of the fig 3.10. Both the 10:1 and 5:1 flyby models exhibit an oscillatory nature on top of the time-varying spiral arms strength, whose nature is similar to those for classical the bulge models. After the pericenter passage, the crest values of these oscillations vary in an approximately similar manner as the spiral arm strength of the classical bulge models. They first increase, reach a peak value and then start decreasing slowly. At the end of the interaction, at 4 Gyr, all the models show similar spiral arm strengths which are equal to the control or isolated model. The isolated models have some initial arms strength $A(p_{max}) = 0.1$ which is because we have taken the initial pseudobulge models after the evolution of bar strength becomes nearly time independent (see the fig. 5.2), and by then these models have grown weak spiral arms. All the flyby models show the same oscillation frequency, irrespective of the flyby mass ratio and pericenter distances. This indicates that these oscillations are intrinsic to the host galaxy and the flyby interactions amplify the amplitudes of these oscillations. Barred galaxies have intrinsic resonances (Binney & Tremaine, 2008a). The oscillations in the effective potential and the positions of equilibrium points near the co-rotation resonances in barred galaxies has been studied by Wu et al. (2016).

3.3.4   Sérsic Index of the Classical Bulge

The evolution of the Sérsic indices of the classical bulges relative to the control isolated model, derived from the two dimensional decomposition of the major galaxy, is shown in fig. 3.11. As in the previous figures, the left and right columns represent flybys of 1:10 and 1:5 mass ratios. The output from the face-on and edge-on bulge-disk fittings are shown in the top and bottom rows respectively. Our classical bulge models have initial Sérsic index values (n) greater than 2. It should be noted that the value of n strongly depends on the surface density of the host galaxy disk i.e. the background of the bulge. For example, the classical bulge without a disk has a Sérsic index of $n = 3.6$ but when the disk is included the face-on fit gives the value $n = 2.1$. However, with the edge-on disk, the Sérsic index remains approximately equal to that without a disk. In our study, we fitted both components of the galaxy simultaneously because that is how it is done in observational studies.

In the face-on fittings, the value of the Sérsic index increases until $t \sim 1.0$ Gyr for all the models (top row of the fig. 3.11). This small increment in the Sérsic index is due to the steepening of the surface density of the inner disk region (Hohl, 1971; Laurikainen & Salo, 2001; Debattista et al., 2006) which contributes to the mass of the spherical/classical bulge. After t=1.0 Gyr, the value of the Sérsic index decreases for all the models. The decrement of the Sérsic index after 1.0 Gyr, which is the time at when the minor galaxy passes the pericenter, is due to the formation of strong spiral arms. The spiral arms reduce the surface density of the outer disk region. For the sake of visualisation, we have plotted the final and initial disk surface densities from the face-on decomposition of unbarred, classical bulge galaxies of 10:1 and 5:1 flybys models in the left and right panels of fig. 3.12 respectively. The density change in the inner and outer disk regions causes the bulge-disk fitting to take some bulge particles into the disk component, thus reducing the value of the Sérsic index. In both flyby models, the change in the Sérsic index just after pericenter passage is the largest for the closest flyby. But it become nearly equal in the end for all the models of the given galaxy mass ratio. In the end, the 5:1 simulations show more change than the 10:1 simulations. Hence it appears that closer and major flybys are more effective in transforming classical bulges into pseudobulges compared to more distant and minor flybys.

However, this apparent decrement in the Sérsic index due to flyby interactions does not really represent the flattening of the classical bulge. This is because if the bulge became more disky or flattened in the z direction, it would be effectively turning into a pseudobulge or it would have a more oval shape. However, we could not detect any such change in the bulge shape in any of the classical bulge models. Fig. 3.13 illustrates this result. It shows the time evolution of the minor to major axis ratio ($q = b/a$) of the classical bulges. The left and right columns represent the 10:1 and 5:1 flyby models respectively. The top and bottom rows show

Fig. 3.11 Evolution of the percentage change in the Sérsic index of the bulge in unbarred galaxies due to flybys. Left and right columns show the flyby simulations of 1/10 and 1/5 mass minor galaxy respectively around the major galaxy with classical bulge. Top and bottom rows represent the outputs from face-on and edge-on galaxy fitting respectively. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\,kpc$, $r_p = 60\,kpc$, and $r_p = 40\,kpc$ respectively.

Fig. 3.12 The left and right panels show the initial and final disk surface densities from face-on decomposition of the unbarred galaxies of 10:1 and 5:1 flyby models respectively. The black solid curves show the initial profiles and red dashed curves, green dash-dotted curves, blue dotted curves, and cyan dash-spaced curves represent the final profiles of the *isolated*, $r_p = 80\,kpc$, $r_p = 60\,kpc$, $r_p = 40\,kpc$ models respectively. The rectangular insets show the 5 times zoom in regions.

the outputs of face-on and edge-on fitting respectively. All the panels show nearly constant axis ratios for the bulges. But the two dimensional decomposition of the face-on galaxy gives a lower Sérsic index which corresponds to the flattening of the bulge. This lowering of the Sérsic index, however, is due to the changing mass distribution in the disk caused by the flyby interaction as discussed earlier in this subsection. Hence, we conclude that classical bulge shapes are not changing due to flyby interactions.

In the edge-on fittings (see the bottom row of the fig. 3.11), in contrast to the face-on fittings, the value of the Sérsic index for all the flyby models is always greater than or equal to the corresponding isolated model. Hence, the flybys are not decreasing the Sérsic index of the classical bulge (or flattening the spherical bulge). Therefore, the real indicator of bulge type/shape is the edge-on decomposition of the galaxy.

3.3.5   Mass of the Classical Bulge

To see whether flybys affect the mass of classical bulges or not, we calculated the bulge mass at time steps of 0.2 Gyr using the integrated magnitudes of the bulges obtained from the output file of GALFIT. The left and right columns of fig. 3.14, show the time evolution of the percentage change in the bulge mass of unbarred, classical bulge galaxies for the 10:1 and 5:1 flyby models

Fig. 3.13 Evolution of the minor to major axis ratio ($q = b/a$) of the classical bulges in unbarred galaxies. Left and right columns show the 10:1 and 5:1 flybys simulations respectively. Top and bottom rows represent the outputs from face-on and edge-on galaxy fitting respectively. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80 \, kpc$, $r_p = 60 \, kpc$, and $r_p = 40 \, kpc$ respectively.

Fig. 3.14 Evolution of the percentage change in the mass of the bulge in unbarred galaxies due to flybys. Left and right columns show the flyby simulations of $1/10$ and $1/5$ mass minor galaxy respectively around the major galaxy with classical bulge. Top and bottom rows represent the outputs from face-on and edge-on galaxy fitting respectively. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\,kpc$, $r_p = 60\,kpc$, and $r_p = 40\,kpc$ respectively.

respectively. The output from face-on and edge-on fittings are shown in the top and bottom rows of the figure respectively. The initial mass of the bulge in the face-on fitting ($0.42 \times 10^{10} M_{\odot}$) is smaller than the edge-on fitting ($0.55 \times 10^{10} M_{\odot}$). The bulge mass from edge-on fitting is much closer to the initial value of $0.6 \times 10^{10} M_{\odot}$. In the face-on fittings, there are clear evolutionary trends which are similar to the evolutionary trends of the Sérsic indices. There is a marginal increase in the bulge masses until the time t=1.0 Gyr of the evolution. This marginal increment is due to the steepening of the surface density of the inner disk region (Hohl, 1971; Laurikainen & Salo, 2001; Debattista et al., 2006) which contributes to the mass of the bulge. Then there is some decrease in the bulge masses for both types of flyby models after 1.0 Gyr of the evolution. This decrease in the bulge mass is due to the formation of strong spiral arms after the flyby interaction. The formation of spiral arms reduces the surface density in the outer region of the disk (see fig. 3.12). Hence, the redistribution of disk particles causes some of the bulge particles to fit into the disk which changes the mass of the bulge in the 2d bulge-disk decomposition of the face-on galaxies. For a given pericenter distance, the 5:1 flybys show more change than the 10:1 flybys. Similarly, the closest pericenter passage shows the more change at a given major to minor galaxy mass ratio. But at the end of the simulations of a given galaxy mass ratio, the change in all the models appear to converge to the same value. On the other hand, edge-on fittings do not show any change in the bulge mass for both the flyby models as seen in the bottom row of the figure. Thus we conclude that flybys do not effect the mass of the classical bulges in the major galaxy.

3.3.6 Angular Momentum of the Classical Bulge

Flybys play an important role in the angular momentum transfer between galaxies. They can spin-up or spin-down the galaxies depending on the configuration of the flyby, such as prograde-prograde, prograde-retrograde, retrograde-prograde, or retrograde-retrograde directions of the galaxies orbits (Bett & Frenk, 2012). But how do the bulges of the galaxies respond to the angular momentum transfer in flybys? To answer this question, we have calculated the z-component of the angular momentum of the bulge and disk within a sphere of radius 5 times the bulge effective radius ($5R_e$). In fig. 3.15, we have shown the time evolution of the bulge to disk angular momentum ratio in unbarred, classical bulge galaxies. The left and right panels of this figure show the 10:1 and 5:1 flyby models respectively. We have plotted only the ratio of the z-component of the angular momenta because all of our model galaxies lie in the x-y plane.

From the two panels of the fig. 3.15, we can clearly see that the ratio of bulge to disk angular momentum is increasing with time in all the models. This is the indication of the angular momentum transfer from the disk to bulge. However, until 2 Gyr, all the flyby models show a gain similar to that of the isolated model. There is a small angular momentum gain in the flyby

Fig. 3.15 Evolution of the bulge to disk angular momentum ratio within a sphere of 5 bulge effective radius ($5R_e$) in unbarred galaxies. Left and right panels show the 10:1 and 5:1 flyby models respectively. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\,kpc$, $r_p = 60\,kpc$, and $r_p = 40\,kpc$ respectively.

models after 2 Gyr but it is not significant enough to clearly distinguish an angular momentum gain in the flyby models compared to the corresponding isolated model. Although it is possible that multiple prograde flybys may add up to a significant transfer of angular momentum to the bulge, it must however be noted that in nature, flybys are not always on prograde orbits. Hence, classical bulges do not gain significant angular momentum in flybys, but instead they could gain angular momentum due to the rotating disk (Pedrosa & Tissera, 2015). The classical bulges in galaxies reside in the deepest part of the potential and there in these models there is no bar type instability in their disks. Hence they do not gain much angular momentum. However, the case is very different when the galaxies are barred as there can be significant angular momentum gain by the bulge (Kataria & Das, 2018).

3.3.7   Evolution of the pseudobulge

As discussed before in the subsection 5.2.2, the existence of the bar and the boxy/peanut pseudobulge makes the 2d decomposition of the galaxy very difficult. Therefore, we have used the kinematic decomposition method for the models with pseudobulges (PG_{PB}) (Abadi et al., 2003). For the kinematic decomposition of the bulge, stellar particles within 15 kpc from the center of the galaxy are used because the bulge resides in the center of the galaxy. We simultaneously fitted the peaks corresponding to the bulge and the disk in the $\varepsilon = L_z/L_c(E)$

Fig. 3.16 An example of two generalized Gaussian fitting (up to the disk peak) to the kinematic decomposition of bulge and disk components of the galaxy with pseudobulge (PG_{PB}). The X-axis shows the ratio of stellar particle angular momentum perpendicular to the disk plane (L_z) and the maximum circular angular momentum ($L_c(E)$) corresponding to stellar energy E. The peak around $\varepsilon = 1.0$ shows the rotation dominated component and the peak away from $\varepsilon = 1.0$ shows the dispersion dominated component. The red filled circles and blue continuous curve show the data points and best fit curve respectively.

distribution plot using two generalized Gaussians (for example see the fig. 3.16). We used the position (ε_{mean}) and full width at half maximum (ε_{fwhm}) of the generalized Gaussian corresponding to the bulge component as the indicator of bulge evolution. The idea is that the ε_{mean} of the bulge will move toward $\varepsilon = 0$ on increasing the random motion of the stellar particles, which represents the dynamical heating of the bulge, and vice-versa for dynamical cooling which is due to increasing ordered motion of the stellar particles. Similarly, the ε_{fwhm} will be larger for bulges that are less distinguishable from the disk compared to small ε_{fwhm} values which represent bulges that are dominant in disks.

The top and bottom rows of fig. 3.17 demonstrate the time evolution of the mean position (ε_{mean}) and full width at half maximum (ε_{fwhm}) of the bulge component in galaxies with pseudobulges (PG_{PB}) respectively. The left and right columns show the outputs from 10:1 and 5:1 flyby models respectively. From the left column of this figure, one can easily interpret that there is no effect of the 10:1 flyby interactions on the pseudobulge of the major galaxy. The time evolution of ε_{mean} and ε_{fwhm} in the 10:1 flyby models are approximately similar to that of the corresponding isolated models. Although these flyby models do show some increase in the ε_{mean} during the closest pericenter passage, it does not last long. Hence, we can conclude that flybys with relatively low mass satellites cannot affect the evolution of the pseudobulge of the host galaxy.

Fig. 3.17 The top and bottom rows show the time evolution of the position (ε_{mean}) and full width at half maximum (ε_{fwhm}) of the generalized Gaussian fitted to the pseudobulge component. Left and right columns represent the flyby simulations of 1/10 and 1/5 mass minor galaxy respectively around the major galaxy with pseudobulge. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\ kpc$, $r_p = 60\ kpc$, and $r_p = 40\ kpc$ respectively.

Fig. 3.18 The time evolution of the ratio of the mean azimuth velocity to mean vertical dispersion ($< v_\phi > / < \sigma_z >$) within 15 kpc. Left and right columns represent the flyby simulations of 1/10 and 1/5 mass minor galaxy respectively around the major galaxy with pseudobulge. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\ kpc$, $r_p = 60\ kpc$, and $r_p = 40\ kpc$ respectively.

In contrast to the 10:1 flyby simulations, the 5:1 flyby simulations show significant deviation from the corresponding control isolated models, as can be seen from the right column of the fig. 3.17. The values of ε_{mean} and ε_{fwhm} decrease relative to the corresponding isolated models after the passage of the minor galaxy. The decrease in ε_{mean} indicates increasing random motion in the bulge i.e the bulge becomes kinematically hotter. On the other hand, decrease in the ε_{fwhm} value indicates that the bulge is becoming more distinct from the disk. These results show that flybys with relatively large mass ratios (such as 1:5) can increase the random motion of stars in the pseudobulge of the host galaxy.

The above mentioned kinematic decomposition of bulge and disk component requires the prior knowledge of the lesser know dark matter halo distribution. Therefore, we have also shown the evolution of the cold and hot component fractions of the galaxy in terms of observable quantities; mean azimuth velocity ($< v_\phi >$) and mean vertical dispersion ($< \sigma_z >$). Fig. 3.18 shows the time evolution of the ratio of the mean azimuth velocity to mean vertical dispersion ($< v_\phi > / < \sigma_z >$) within 15 kpc. The left and right panels of the figure represent

the 10:1 and 5:1 mass ratio flyby simulations respectively. The left panel clearly shows that the 10:1 flyby models follow the same trend as that of the control isolated model. There is no effect of 10:1 flyby interactions on the evolution of the pseudobulge. But, the 5:1 simulations show a different trends from the control isolated model as can be seen from right panel of the figure. There is more vertical dispersion in the flyby model than the isolated model. This supports the previous results that the flyby interactions with larger mass ratios can heat the pseudobulge more significantly than smaller mass ratios in minor flyby interactions.

3.3.8   Mass and Angular Momentum of the Pseudoulge

In the previous subsection, we have discussed the effects of flyby interactions on the kinematic heating and separability of a pseudobulge from the disk. In this section we investigate its effect on the mass and angular momentum of the pseudobulge. To obtain the mass and angular momentum of the bulge using kinematic decomposition, we have used a widely accepted fixed value of $\varepsilon = 0.7$ as a separator of the bulge and disk particles (Peebles, 2020a). The particles with $\varepsilon < 0.7$ are considered as bulge particles and particles with $\varepsilon \geq 0.7$ are considered as disk particles.

The time evolution of the bulge to total stellar mass ratio (B/T) and the disk to bulge angular momentum ratio ($L_{(z,b)}/L_{(z,d)}$) within 15 kpc (the maximum radius for kinematic decomposition) are shown in the top and bottom rows of fig. 3.19 respectively. The left and right columns represent the flybys of 1/10 and 1/5 mass ratios for a major galaxy with a pseudobulge. Both panels of the left column show the slow increase in the mass and angular momentum of the bulge in flybys relative to the control isolated model. But after the flyby ends, all the models converge to values similar to that of the isolated model. Thus, the 10:1 flyby models do not show any net effect of the interaction on the mass and angular momentum of the pseudobulge. Hence, flyby interactions with relatively low mass satellites show only small changes in mass and angular momentum, and the effect disappears after the satellite has moved away.

On the other hand, the effect of 5:1 flyby interactions are very significant, as can be seen from comparing the adjacent columns of fig. 3.19. The mass and angular momentum gained by the pseudobulge remains always larger than the control isolated model. At time t~2 Gyr, there is approximately 20% angular momentum gain in flyby interactions relative to the isolated evolution. The trends of mass and angular momentum gain are approximately similar. Therefore, there is a continuous transfer of mass from the disk to the pseudobulge. This is a clear indication of the kinematic heating of the pseudobulge and bulge growth. Hence, relatively massive satellites can help the pseudobulges in flybys to gain mass and angular momentum from the disk. It is possible that the effect of multiple flyby interactions can strongly affect the evolution of psudobulges.

Fig. 3.19 The top and bottom rows show the time evolution of the bulge to total mass ratio (B/T) and bulge to disk angular momentum ratio ($L_{(z,b)}/L_{(z,d)}$) within 15 kpc respectively. The left and right columns represent the flyby simulations of $1/10$ and $1/5$ mass minor galaxy respectively around the major galaxy with pseudobulge. The red circle solid curves, green down triangle solid curves, blue upper triangle solid curves, and cyan diamond solid curves represent the models *isolated*, $r_p = 80\,kpc$, $r_p = 60\,kpc$, and $r_p = 40\,kpc$ respectively.

3.3.9   Effect of Flyby on the Buckling of the Bar

We have previously discussed the response of pseudobulges to flyby interactions with 10:1 and
5:1 mass ratios. The origin of the pseudobulge discussed in this chapter is the buckling of the
bar. An important question is then how does bar buckling respond to flyby interactions? This is
a relevant question as bar buckling modifies the kinematics, rendering the system kinematically
hotter. To understand the effect of the flyby interactions on bar buckling we have simulated two
5:1 mass ratio flyby interactions, one starting before the first peak in the bar strength and other
starting before the second peak in the bar strength (see fig. 5.2). The choice of 5:1 mass ratio is
made simply because the 10:1 mass ratio does not seem to effect pseudobulges as much as the
larger mass ratio.

To quantify the strength of the buckling of the bar, we have adopted some commonly used
physical quantities in the literature (Ciambur et al., 2017; Łokas, 2019a). In fig. 3.20, we
have shown the time evolution of absolute value of mean vertical velocity($|< v_z >|$), mean
vertical dispersion ($< \sigma_z >$), ratio of the mean azimuth velocity to mean vertical dispersion
($< v_\phi > / < \sigma_z >$), and the root mean square value of the vertical positions (z_{rms}) in the top to
bottom panels respectively. All these quantities are calculated within the 7.5 kpc radius of the
galaxies (half of that used in the kinematic decomposition) which is large enough to include
the bar region throughout the evolution. The red circle solid curve, green down triangle solid
curve, and blue upper triangle solid curve represent the control isolated model, flyby stating at
$t_s = 0$ Gyr from control model, and flyby stating at $t_s = 1$ Gyr from control model respectively.

From the top panel of the fig. 3.20, one can see that all the models show a sharp peak at the
same time in the evolution of $|< v_z >|$. These peaks occur at the time of the second peak in
the bar strength. The peak in the evolution of mean v_z signifies the fraction of stars going out
of the disk plane. The height of the peak quantifies the magnitude of bar bending out of the
disk plane. Though the bending is not strong but it seems that the control isolated model bends
the most and flyby interactions reduce the bar bending. The second panel from the top shows
the evolution of mean σ_z which quantifies the the vertical height. All the models show similar
evolution of $< \sigma_z >$. This result can be further confirmed from the evolution of the other two
quantities. The evolution of $< v_\phi > / < \sigma_z >$, and z_{rms} in the flyby models follows a similar
trend as that of the control isolated model. Hence, flyby interactions do not seem to affect the
time and strength of the bar buckling in our galaxy model.

3.3.10   Effect of Numerical Resolution on Our Results

All the results of the flyby interactions, discussed in this chapter, are relative to the control
isolated model. The flyby interactions include secular evolution of the galaxies which can

Fig. 3.20 The time evolution of absolute value of mean vertical velocity($| < v_z > |$), mean vertical dispersion ($< \sigma_z >$), ratio of the mean azimuth velocity to mean vertical dispersion ($< v_\phi > / < \sigma_z >$), and root mean square value of vertical coordinates (z_{rms}) in the top to bottom panels respectively. The red circle solid curve, green down triangle solid curve, and blue upper triangle solid curve represent the *isolated*, $r_p = 40 \ kpc$ starts at $t_s = 0$ Gyr, and $r_p = 40 \ kpc$ starts at $t_s = 1$ Gyr models respectively.

Fig. 3.21 Effect of numerical resolution on the evolution of the percentage change in the disk scale height to disk scale radius ratio in 10:1 flyby model of classical bulge galaxy. The green diamond solid curve represents the simulation of $r_p = 40 \, kpc$ model with twice as many particles as red circle solid curve. Both the curves show nearly similar trends.

only be removed by comparing with the evolution of an isolated galaxy. On the other hand, numerical simulations always suffer from the discreteness of particle distribution. Some effects of this discreteness on the results of flyby simulations can be removed by subtracting the secular evolution from the evolution in interactions. This is the reason why we compared our results with the corresponding isolated models. We chose the number of particles for our simulation using an analytical study of two body relaxation (Power et al., 2003). We also performed a few high resolution simulations with twice as many particles as given in the Table 5.1. We find that the trends as discussed in the results section (section 7.5) are consistent with these high resolution simulation. For comparison, in fig. 3.21, we have shown the time evolution of the percentage change in the disk scale height to disk scale radius ratio in the 10:1 flyby model of a classical bulge galaxy with respect to the corresponding isolated model. The green diamond solid curve represents the simulation of $r_p = 40 \, kpc$ model with twice as many particles as red circle solid curve. Both curve show more or less same evolutionary trends. This indicates that our results are consistent with high resolution.

3.4   Implications

In the previous sections, we have presented a qualitative and quantitative study of the effect of flyby interactions on the disks, spiral arms, classical bulges (in unbarred galaxies) and pseudobulges in galaxies. The results of our simulations are very significant for observations of interacting galaxies because in our local universe the rate of flybys is larger at lower redshifts (z<2) (Sinha & Holley-Bockelmann, 2012a). In the following paragraphs we discuss the implications of our study for observations of flyby interactions in the local universe.

The gravitational force of a flyby produces strong perturbations in the outer disk regions of the major galaxy. As a result, significant amounts of stellar mass is pulled out from the disk in the form of spiral arms/ tidal tails. This results in the decrease of the disk scale radius (R_s) and the increase of the disk scale height (z_0). Although the increase in z_0 is small, the ratio of disk scale height to disk scale radius ($\frac{z_0}{R_s}$) shows a significant increase (Reshetnikov & Combes, 1996, 1997a). Since a galaxy experiences multiple flybys during its evolution, the resultant vertical thickening or heating of the host galaxies will be significant and perhaps comparable to that produced by minor mergers.

Flyby interactions stimulate the formation of strong spiral arms in the host galaxies just after the pericenter passage. However, the maximum strength of the spiral arms attained in the flyby does not last. It decays slowly with time and all the flyby models show nearly the same strength in their spiral arms after 4 Gyrs. Our N-body simulations thus show the slowly decaying nature of the spiral arms. This supports the idea that spiral arms are not static density waves as discussed in observations (Masters et al., 2019). These results will be very helpful in understanding the nature of spiral arms in interacting and isolated galaxies. The spiral arms induced by a closely bound satellite galaxy may survive for a long period of time because a bound satellite will always perturb its host galaxy. The effect of multiple satellite galaxies orbiting around their host on different orbital configurations can give a detailed insight into the origin of the spiral structure.

Strong spiral arms are formed in both the unbarred, classical bulge galaxies and the pseudobulge galaxies. But in contrast to the unbarred, classical bulge galaxies, the flyby induced spiral arms in the pseudobulge host galaxies show oscillations on the top of the time-varying strength of the spiral arms. These oscillations in the spiral arms strength A_2/A_0 could be due to existing small amplitude transients near the resonances associated with the bar (Binney & Tremaine, 2008a; Wu et al., 2016), which have been detected in earlier simulations of barred galaxies. The oscillations may be amplified by the energy input due to the flyby interaction. These transient spiral waves may have important implications for star formation and are triggered by the flyby. A good way to test this is by comparing the star formation rates in flybys with host galaxies that have bars and those that do not have bars.

Our overall results of flybys with classical bulges suggests that the inner regions of such galaxies are very stable. Neither the bulge morphology nor the bulge angular momentum changes significantly during flybys. The photometric bulge-disk decomposition of the face-on unbarred, classical bulge galaxies shows a decrease in the mass and Sérsic index of the classical bulge. This apparent decrease in the Sérsic index of the classical bulge can be mistakenly considered as evidence for the transformation of the classical bulge into the pseudobulge. So if we take the Sérsic index as a parameter for distinguishing between classical and pseudobulges, a significant fraction of classical bulges will be mistakenly classified as pseudobulges in face-on galaxies. As a consequence, this classification will provide more weight to the rotation dominated universe and challenge the hierarchical nature of the structure formation (Fisher & Drory, 2011a). So it is clearly important to examine the edge-on decompositions of galaxies whenever possible for Sérsic index studies of bulges.

Our simulations also show that although the pseudobulges are supported by the ordered motion of the stars, they are very stable against the small perturbations in the outer disk regions of the galaxies. In the 10:1 mass ratio flybys, the pseudobulges do not show any noticeable change except during pericenter passage. There are signatures of dynamical heating, mass growth and angular momentum gain only in the 5:1 mass ratio flybys. Thus only similar mass flybys or large mass ratio minor flybys ($>$1:5) are efficient enough to perturb the evolution of pseudobulges.

Our simulations do not account for the presence of gas in the galaxies. Therefore our findings are valid only for dry or gas poor galaxies. To make a rough estimate of the difference that gas inflow during a flyby would make to the bulge mass, consider the following. Let us assume that the central gas mass fraction of the major galaxy is 5-10% of the total stellar mass and the star formation efficiency (SFE) is $\sim$10%. During the evolution if the gas is converted to stars, it will contribute 3-6% of the bulge mass in the center of the galaxy during the flyby. Hence, a gas rich galaxy with a high star formation rate (SFR) may show an increase in bulge mass during such flyby interactions but it will not significantly affect our results.

3.5   Conclusions

We have simulated 10:1 and 5:1 mass ratio flyby interactions of an unbarred, classical bulge galaxy and a barred galaxy with a pseudobulge. We study the effect of the flyby on the major galaxy only. The galaxies are on prograde-prograde orbits, with pericenter distances (distance of closest approach) varying from 40 to 80 kpc. We evolved the flyby for 4 Gyrs and then quantified the evolution of the bulge, disk and spiral arms of the major galaxy using bulge-disk decomposition and Fourier analysis techniques. The main findings of our study are as follows.

1. The disk scale radius (R_s) of the galaxies decreases in the flyby interactions. The change in the disk scale radius is strongly correlated with the pericenter distance and mass ratio of the galaxies. For a given mass ratio, the close flyby shows the most decrease but for a given pericenter distance, the major flyby (5:1) shows the most decrease. Due to presence of the spiral arms, this change is more pronounced in face-on decomposition than egde-on decomposition. The final disk scale radius seems to converge to the same value for all pericenter distances for a given flyby mass ratio.

2. The absolute disk scale height (z_0) of the galaxies is marginally affected by the flyby interactions. There is a maximum increase of $\sim 4\%$ in the disk scale height for our set of simulations. However, the ratio of disk scale height to disk scale radius (z_0/R_s) depends significantly on the pericenter distance. The galaxies with smaller pericenter distance show larger changes in this ratio, which is similar to observations of interacting galaxies (Reshetnikov & Combes, 1996, 1997a). But our simulations also show that this is the effect of reducing disk scale length.

3. The minor flyby interactions reduce the disk size but at the same time result in the formation of the grand design spiral arms. The simulations of unbarred, classical bulge galaxies suggests that grand design spiral arms can form in minor flyby interactions when the major galaxy has a concentrated bulge and/or low surface density disk. Both features prevent bar formation in the disk. Hence, grand design spiral galaxies form only in flyby interactions of bar-stable, unbarred galaxies with satellite galaxies.

4. All the models show that the spiral arm strength increases with decreasing pericenter distance passage. However, at the end of the flyby simulation (at 4 Gyrs), all the flyby models show approximately similar spiral arms strength values for a given galaxy mass ratio model, indicating that the final effect does not depend on pericenter distance. The barred, pseudobulge galaxies show small oscillations in the strength of the spiral arms on top of the overall spiral arm strength. The amplitude of these oscillations increases with decreasing pericenter distance. The frequency of these oscillations is approximately independent of the pericenter distances and galaxy mass ratios.

5. The classical bulge of the major galaxy remains generally unaffected by the flyby interactions. In the face-on bulge-disk decomposition, the apparent change in the bulge Sérsic index is instead due to the steepening of the inner disk surface density and the formation of spiral arms during the flyby. But the edge-on fittings and visualizations of the bulge indicates that the bulge morphology remains unchanged. The kinematic decomposition of the pseudobulges shows some heating and mass growth in the 5:1 simulations. This growth comes at the cost of the disk heating/thickening.

6. There is only a very minute increment in the angular momentum of the classical bulges. This small change is not due to the flyby interactions but due to the rotational motion of the disk particles. The change in the angular momentum of the classical bulges is nearly similar for all the flyby models and isolated models. The evolution of the classical bulges in dry (of gas poor) galaxies remain unaffected of flyby interactions. Pseudobulges show around 20% gain in the bulge angular momentum after 2 Gyr relative to the secular evolution. But most of it comes from the conversion of disk particles to bulge particles by vertical heating of the disk.

7. The tidal force of the orbiting satellite can induce bar formation in the host galaxy (Lang et al., 2014; Łokas, 2018). Tidally induced bars can also buckle with different different strength (Łokas, 2019a). In our study we find that flyby interactions affect neither the time nor the strength of bar buckling in our galaxy models.

4

Excitation of Vertical Breathing Motion in Disc Galaxies by Tidally-induced Spirals in Fly-by Interactions

4.1 Introduction

The second data release from the *Gaia* mission (hereafter *Gaia* DR2) has revealed the presence of large-scale bulk vertical motions ($\sim 10\,\mathrm{km\,s^{-1}}$ in magnitude) and the associated bending and breathing motions for stars in the Solar vicinity and beyond (Gaia Collaboration et al., 2018). The presence of such breathing motions, i.e., stars on both sides of the Galactic mid-plane moving coherently towards or away from it, has also been reported in various past Galactic surveys, for example, the SEGUE (Sloan Extension for Galactic Understanding and Exploration) survey (Widrow et al., 2012), the LAMOST (Large Sky Area Multi-Object Fibre Spectroscopic Telescope) survey (Carlin et al., 2013), and the RAVE (Radial Velocity Experiment) data (Williams et al., 2013). The existence of such non-zero bulk vertical motions in the Milky Way raises questions about the plausible driving mechanism(s), since, in an

axisymmetric potential, the bulk radial and vertical motions should be zero (e.g. Binney &
Tremaine, 2008b).

Much of the understanding of the excitation of breathing motions in Milky Way-like galaxies
have been gleaned from numerical simulations. Using semi-analytic models and test-particle
simulations, Faure et al. (2014) was the first to show that a strong spiral can drive large-scale
vertical motions ($|\langle v_z \rangle| \sim 5 - 20\,\mathrm{km\,s^{-1}}$). The amplitude of such breathing motion increases at
first with height from the mid-plane, and then starts to decrease after reaching its maximum
value at a certain height. Also, using self-consistent N-body simulation, Debattista (2014)
showed that a *vertically-extended* spiral feature can drive strong large-scale breathing motions,
with amplitude increasing monotonically from the mid-plane. The relative sense of these bulk
motions, whether compressing or expanding, changes across the corotation resonance (hereafter
CR) of the spiral (Faure et al., 2014; Debattista, 2014). Furthermore, using a *self-consistent*,
high-resolution simulation with star formation, Ghosh et al. (2022a) studied the age-dependence
of such vertical breathing motions excited by spiral density waves. They showed that, at fixed
height, the amplitude of such vertical breathing motion decreases with stellar age. They showed
a similar age-variation in the breathing amplitude in the *Gaia* DR2, thereby supporting the
scenario that the breathing motion of the Milky Way might well be driven by spiral density
waves (Ghosh et al., 2022a). Instead, Monari et al. (2015) showed that a stellar bar can also
drive such breathing motion in disc galaxies. However, the resulting amplitudes of the breathing
motions are small ($|\langle v_z \rangle| \sim 1\,\mathrm{km\,s^{-1}}$) when compared to the spiral-driven breathing amplitudes.

In the Lambda cold dark matter (ΛCDM) paradigm of hierarchical structure formation,
galaxies grow in mass and size via major mergers and/or multiple minor mergers, and cold gas
accretion (White & Rees, 1978a; Fall & Efstathiou, 1980). During the evolutionary phase, a
galaxy also experiences multiple tidal interactions with satellites and/or passing-by companion
galaxies. The frequency of such fly-by encounters increases at lower redshifts (e.g., see Sinha
& Holley-Bockelmann, 2015b), and their cumulative dynamical impact on the morphology as
well as on the dynamics of the host galaxies can be non-negligible (e.g., see An et al., 2019b).
Fly-by encounters can excite an $m = 2$ bar mode (e.g., see Noguchi, 1987; Łokas et al., 2016;
Martinez-Valpuesta et al., 2017; Ghosh et al., 2021), off-set bars with a one-arm spiral (e.g,
Pardy et al., 2016), and an $m = 1$ lopsidedness in the stellar disc (Bournaud et al., 2005c;
Mapelli et al., 2008; Ghosh et al., 2022b). They can also trigger star formation (e.g., Duc
et al., 2018b), and if the gas inflow towards the centre is exceedingly large, this can lead to a
starburst (Mihos & Hernquist, 1994b) as well as triggering of AGN activity (Combes, 2001b).
Furthermore, the role of fly-bys has been investigated in the context of forming warps (Kim
et al., 2014b; Semczuk et al., 2020), disc heating and disc thickening (Reshetnikov & Combes,
1997b; Kumar et al., 2021), tidal bridges and streams (e.g., see Toomre & Toomre, 1972; Duc &

Renaud, 2013b), altering the galaxy spin (Choi & Yi, 2017b), and the evolution of classical and pseudo-bulges (Kumar et al., 2021). Our Galaxy has also experienced such a tidal interaction with the Sagittarius (Sgr) dwarf galaxy (e.g., see Majewski et al., 2003). Recent studies have indicated that such a tidal interaction could excite a 'snail-shell' structure (phase-space spiral), bending motions in the Solar neighbourhood (for details see, e.g., Widrow et al., 2014; Antoja et al., 2018b; Gaia Collaboration et al., 2018).

A tidal encounter with another galaxy can excite spiral features, as was first proposed in the seminal work of Holmberg (1941). Later, pioneering numerical work of Toomre & Toomre (1972) showed that a tidal interaction can excite tidal tails, bridges, and spiral features in the disc of the host galaxy for a wide variety of orbital configurations. Following that, numerical simulation has become an indispensable tool to study the dynamical effect of galaxy interactions. Several past studies attempted to understand the role of tidal encounters in the context of excitation of spirals as well as to understand the longevity and nature of the resulting spirals (e.g. see Sundelius et al., 1987; Donner & Thomasson, 1994; Salo & Laurikainen, 2000; Dobbs et al., 2010; Pettitt & Wadsley, 2018). Furthermore, a recent study by Pettitt et al. (2017) investigated star formation and the properties of interstellar medium in tidally-induced spirals.

Tidal interactions can also induce vertical distortions and oscillations in the disc of the host galaxy (e.g., see Hunter & Toomre, 1969; Araki, 1985; Mathur, 1990; Weinberg, 1991; Vesperini & Weinberg, 2000; Gómez et al., 2013; Widrow et al., 2014; D'Onghia et al., 2016). Widrow et al. (2014) proposed a dynamical scenario where a satellite galaxy, while plunging into the disc, can excite both bending and breathing motions. Interestingly, such tidal interactions also excited a strong spiral response within the disc in their model (see Fig. 10 there). Therefore, it is still unclear whether the tidal interactions are 'directly' responsible for driving breathing motion, or the tidally-induced spirals are driving the breathing motions.

We aim to test this latter hypothesis in detail in this chapter. We study a set of N-body models of galaxy fly-by interactions while varying the orbital parameters. We investigate the generation of the spiral features due to such fly-by encounters, and quantify the nature and longevity of such spirals in different fly-by models. We closely follow the generation of the vertical breathing motions and their subsequent evolution. In particular, we look for evidence that the generation and evolution of the vertical breathing motions is correlated with the temporal evolution of the tidally-induced spirals.

The rest of the chapter is organized as follows. Section 4.2 provides the details of the simulation set-up and the fly-by models. Section 4.3 presents the results of the tidally-induced spirals, the density wave nature of spirals, and the temporal evolution of their strength. Section 4.4 measures the properties of the vertical breathing motions driven by these tidally-induced spirals.

Table 4.1 Key galaxy parameters for the equilibrium models of the host and the perturber galaxies.

Galaxy	$M^{(1)}$ $(\times 10^{12} M_\odot)$	$\lambda^{(2)}$	$c^{(3)}$	$f_{\rm disc}^{(4)}$	$f_{\rm bulge}^{(5)}$	$j_{\rm d}^{(6)}$	$R_{\rm d}^{(7)}$ (kpc)	$z_0^{(8)}$ (kpc)	$N_{\rm halo}^{(9)}$ $(\times 10^6)$	$N_{\rm disc}^{(10)}$ $(\times 10^6)$	$N_{\rm bulge}^{(11)}$ $(\times 10^6)$	$N_{\rm tot}^{(12)}$ $(\times 10^6)$
Host	1.2	0.035	10	0.025	0.005	0.03	3.8	0.38	2.5	1.5	1	5
Perturber	0.24	0.035	11	0.01	0.002	0.01	1.95	0.195	0.5	0.3	0.2	1

(1) total mass (in $M_\odot$); (2) halo spin; (3) halo concentration; (4) disc mass fraction; (5) bulge mass fraction; (6) disc spin fraction; (7) disc scale length (in kpc); (8) disc scale height (in kpc); (9) total DM halo particles; (10) total disc particles; (11) total bulge particles; (12) total number of particles used.

Section 6.6 discusses a few limitations of this work while section 4.6 summarizes the main findings of this work.

4.2 Simulation set-up of galaxy Fly-by models

To motivate our study, we construct a set of *N*-body models of galaxy fly-bys where the host galaxy experiences an unbound interaction with a perturber galaxy. The mass ratio of the perturber and the host galaxy is set to 5:1, and is kept fixed for all the models considered here. A prototype of such a galaxy fly-by model is already presented in Kumar et al. (2021). Here, we construct a suite of fly-by models varying the orbital configuration (e.g., angle of interaction, orientation of the orbital spin vector). The details of the modelling and the simulation setup is discussed in Kumar et al. (2021). For the sake of completeness, here we briefly mention the equilibrium model of the galaxies as well as the orbital configurations of the galaxy interaction.

4.2.1 Equilibrium models

The initial equilibrium model of each galaxy (host and the perturber) consists of a classical bulge, a stellar disc, and a dark matter (hereafter DM) halo. Each of the galactic components is treated as live, thereby allowing them to interact with each other. The DM halo is assumed to be spherically symmetric, and is modelled with a Hernquist density profile (Hernquist, 1990) of the form

$$\rho_{\rm dm}(r) = \frac{M_{\rm dm}}{2\pi} \frac{a}{r(r+a)^3}, \qquad (4.1)$$

where $M_{\rm dm}$ and a are the total mass and the scale radius of the DM halo, respectively. The scale radius of the Hernquist halo is related to the concentration parameter 'c' of NFW DM halo (Navarro et al., 1996). For an NFW DM halo with mass $M_{200} = M_{\rm dm}$, this relation is given by

the following equation,

$$a = \frac{r_{200}}{c} \sqrt{2 \left[\ln\left(1+c\right) - \frac{c}{(1+c)} \right]}, \tag{4.2}$$

where r_{200} represents the radius of an NFW halo[1], and the mass within this radius is defined as M_{200}. The classical bulge is also assumed to be spherically symmetric and is modelled with another Hernquist density profile (Hernquist, 1990) of the form

$$\rho_b(r) = \frac{M_b}{2\pi} \frac{b}{r(r+b)^3}, \tag{4.3}$$

where M_b and b represent the total bulge mass and the bulge scale radius, respectively. The initial radial surface density of the stellar disc follows an exponential fall-off and has a $sech^2$ profile along the vertical direction, thereby having the form

$$\rho_d(R,z) = \frac{M_d}{4\pi z_0 R_d^2} \exp\left(-\frac{R}{R_d}\right) sech^2\left(\frac{z}{z_0}\right), \tag{4.4}$$

where M_d is the total mass, R_d is the exponential disc scale length, and z_0 is the scale height. The corresponding values of the structural parameters, used to model the host as well as the perturber galaxy, are listed in Table 5.1.

The equilibrium models for the host as well as the perturber are generated using the publicly distributed code GALIC (Yurin & Springel, 2014). This code uses elements of the Schwarzschild's method and the made-to-measure method to search for a stable solution of the collisionless Boltzmann Equation (CBE) for a set of collisionless stellar particles, initialized by some predefined analytic density distribution functions (for details, see Yurin & Springel, 2014). A total of 5×10^6 particles are used to model the host galaxy whereas a total of 1×10^6 particles are used to model the perturber galaxy. The number of particles used to model each of the galaxy components of the host and the perturber galaxy are also listed in Table 5.1. The stellar particles have gravitational softening $\varepsilon = 20\,pc$ while the DM halo particles have $\varepsilon = 30\,pc$. Fig. 4.1 shows the corresponding radial profiles of the circular velocity (v_c) and the Toomre Q parameter for the host and the perturber galaxy at $t = 0$.

4.2.2 Set-up of galaxy fly-by scenario

To simulate the unbound galaxy fly-by scenario, we place our galaxy models on a hyperbolic orbit with eccentricity, $e = 1.1$ so that the orbit of the perturber galaxy remains unbound

---

[1]It is defined as the radius from the centre of the halo inside which the mean density is 200 times the critical density of the Universe.

Fig. 4.1 Radial profiles of the circular velocity (v_c) and the Toomre Q parameter are shown for equilibrium models of the host (left-hand panels) and the perturber galaxy (right-hand panels). In the top panels, the blue line denotes the contribution of the stellar disc while the DM halo contribution is shown by the green line. The bulge contribution is shown by the cyan solid line whereas the red line denotes the total/net circular velocity.

Table 4.2 Key orbital parameters for the galaxy fly-by models.

Model[a]	r_p[b] (kpc)	i°[c]	t_p[d] (Gyr)	t_{enc}[e] (Gyr)	T_p[f]
RP40i00pro	53.09	0	0.88	0.1298	-4.1348
RP40i30pro	52.34	30	0.85	0.1272	-4.1162
RP40i60pro	52.14	60	0.85	0.1265	-4.1113
RP40i90pro	52.23	90	0.85	0.1267	-4.1133
RP40i00ret	53.07	0	0.85	0.1298	-4.1342

(a) Galaxy fly-by model; (b) pericentre distance (in kpc); (c) orbital angle of interaction (in degree); (d) time of pericentre passage (in Gyr); (e) encounter time (in Gyr); (f) tidal parameter.

throughout the interaction. We avoid choosing a parabolic orbit as the dynamical friction of the host galaxy decays the orbit of the perturber galaxy and puts the perturber galaxy on a bound elliptical orbit. Our choice of hyperbolic orbit avoids a bound fly-by interaction. We place the galaxies at an initial separation of 255 kpc before the start of the simulation. For different models with different orbital configurations, we vary the distance of their closest approach (the pericentre distance) assuming the two-body Keplerian orbit. For further details of the orbital configuration and the geometry of the unbound fly-by scenario, the reader is referred to Kumar et al. (2021). A total of five such galaxy fly-by models are used for this study. The angle of interaction, and the pericentre distance for these models are listed in Table 4.2.

All the simulations are run using the publicly available code GADGET-2 (Springel et al., 2001a; Springel, 2005a,b) for a total time of 6 Gyr, with a tolerance parameter $\theta_{\text{tol}} = 0.5$ and an integration time-step 0.4 Myr. The maximum error in the angular momentum of the system is well within 0.15 percent throughout the evolution for all models considered here.

Following Di Matteo et al. (2007), we also calculate the encounter time (t_{enc}), and tidal parameter (T_{p}) for the host galaxy by using

$$t_{\text{enc}} = \frac{r_{\text{p}}}{v_{\text{p}}}, T_{\text{p}} = \log_{10}\left[\frac{m_{\text{per}}}{M_{\text{host}}} \left(\frac{R_{\text{d}}}{r_{\text{p}}} \right)^3 \right], \tag{4.5}$$

where r_{p} is pericentre distance and v_{p} is the corresponding relative velocity. M_{host} and m_{per} are the masses of the host and the perturber galaxy, respectively. The corresponding values of t_{enc}, and T_{p} for all the models consider here, are listed in Table 4.2. We note that the quantities mentioned here will be in units of the dimensionless parameter 'h', defined via the Hubble constant ($H_0 = 100h$ km s^{-1} Mpc^{-1}) and can be scaled to observed values.

Each model is referred as a unique string given by '[PERICENTRE DISTANCE][ANGLE OF INTERACTION][ORBITAL SPIN]' where [PERICENTRE DISTANCE] denotes the pericenter distance, obtained using the standard two-body formalism. [ANGLE OF INTERACTION] denotes the angle at which the perturber encounters with the host galaxy while [ORBITAL SPIN] denotes the orbital spin vector ('pro' for prograde and 'ret' for retrograde orbits). We follow this scheme of nomenclature throughout the chapter. As an example, RP40i30pro denotes a fly-by model where the perturber galaxy interacts with the host galaxy at an angle of 30° in a prograde orbit, and the calculated pericentre distance between these two galaxies is 40 kpc, obtained by using the standard two-body Keplerian orbit.

Fig. 4.2 Top: the density distribution of the stars of the host galaxy in the face-on projection, at four different times after the pericentre passage, for the model *RP*40*i*00*pro*. White solid lines show the contours of constant surface density. A strong spiral feature is excited in the disc of the host galaxy after it experiences a tidal encounter with the perturber galaxy. Bottom: the corresponding distribution of the residual surface density ($\tilde{\Sigma}(R, \phi)$), calculated using Eq. 4.6, at the same four different times. The sense of rotation is towards increasing ϕ.

4.3   Quantification of tidally-induced spirals in galaxy fly-bys

Here, we investigate the excitation of spiral structure in the host galaxy as a consequence of the tidal interaction with the perturber galaxy. We first study the model *RP*40*i*00*pro*, where the host galaxy experiences an in-plane, unbound fly-by interaction with the perturber galaxy (with mass 1/5th that of the host galaxy). During the pericentre passage, the perturber exerts a strong tidal pull on the host galaxy. Fig. 4.2 (top panels) shows the face-on density distribution of the stars of the host galaxy at four different times, after the tidal interaction occurs. A visual inspection reveals that after the interaction happens, a strong spiral feature is excited in the outer parts of the host galaxy (e.g., at $t = 2$ Gyr). The spiral features are also seen at later times. However, by the end of the simulation ($t = 6$ Gyr), there is no discernible, strong spiral features left in the host galaxy. To study this trend further, we calculate the residual surface density

$(\tilde{\Sigma}(R,\phi))$ in the (R,ϕ)-plane using

$$\tilde{\Sigma}(R,\phi) = \frac{\Sigma(R,\phi) - \Sigma_{\mathrm{avg}}(R)}{\Sigma_{\mathrm{avg}}(R)}, \tag{4.6}$$

where $\Sigma_{\mathrm{avg}}(R)$ is the azimuthally-averaged surface density of the disc at radius R. This is shown in Fig. 4.2 (bottom panels). As seen clearly, after the interaction happens, a strong, coherent spiral feature, denoted by the presence of a periodic over- and under-density, is excited in the outer region ($R \geq 10\,\mathrm{kpc}$) of the host galaxy. At the end of the simulation ($t = 6\,\mathrm{Gyr}$), the corresponding residual density distribution does not exhibit any coherent spiral structure in the disc of the host galaxy.

4.3.1   Strength and temporal evolution of spirals

In the previous section, we have shown that a tidal interaction with a perturber galaxy excites a prominent spiral feature in the outer disc region of the host galaxy for the model *RP40i00pro*. Next, we quantify the strength of the spiral and follow its temporal evolution. For this, we first calculate the radial variation of the Fourier moment of the surface density of the stellar particles of the host galaxy using

$$A_m/A_0(R) = \left| \frac{\sum_j m_j e^{im\phi_j}}{\sum_j m_j} \right|, \tag{4.7}$$

where A_m is the coefficient of the mth Fourier moment of the density distribution, m_j is the mass of the jth particle (e.g., see Kumar et al., 2021, 2022a). Fig. 4.3 shows the corresponding radial variation of the $m = 2$ Fourier moment at $t = 2\,\mathrm{Gyr}$ for the model *RP40i00pro*. In the outer disc regions, there are less particles when compared to the inner disc regions. Therefore, using a *linearly-spaced* radial binning when calculating the Fourier coefficient (using Eq. 7.1) introduces noise in the calculation for the outer regions. To avoid that, we employ a logarithmic binning in the radial direction. As seen clearly, in the outer parts ($R \geq 10\,\mathrm{kpc}$), the values of the coefficient A_2/A_0 are non-zero, indicating the presence of a strong spiral structure. Are these tidally-induced spirals mostly confined to the disc mid-plane or are they vertically-extended? To investigate this further, we calculate the radial variation of the same Fourier coefficient A_2/A_0, but for stars in different vertical layers of thickness 400 pc. The resulting radial variations are also shown in Fig. 4.3. As seen clearly from Fig. 4.3, the Fourier coefficient A_2/A_0 shows non-zero values even for stars at the largest heights from the mid-plane ($|z| = [0.8, 1.2]\,\mathrm{kpc}$). Also, at a certain radius R within the extent of the spirals, the values of the Fourier coefficient A_2/A_0 decreases monotonically as one moves farther away from the mid-plane. We checked

Fig. 4.3 Radial variation of the Fourier coefficient of the $m = 2$ Fourier component (normalised by the $m = 0$ component) is shown at $t = 2\,\text{Gyr}$ for the model *RP40i00pro* (see the black solid line). The same quantity is then measured for stars in different vertical layers (as indicated in the legend).

this variation of the Fourier coefficient A_2/A_0 with height at other time-steps as well, and found that this trend remains generic whenever the tidally-induced spirals are strong in the disc of the host galaxy. This demonstrates that the tidally-induced spirals in our model *RP40i00pro* is *vertically-extended*, similar to what was reported in Debattista (2014) and Ghosh et al. (2022a).

To further investigate the spatio-temporal evolution of the tidally-induced spirals in model *RP40i00pro*, we calculate the Fourier coefficient $A_2/A_0(R)$ at different radial locations for the whole simulation run-time (a total of $6\,\text{Gyr}$). This is shown in Fig. 4.4. The tidal interaction with the perturber excites a strong spiral feature in the disc of the host galaxy after $t \sim 1.1\,\text{Gyr}$ or so. These spirals remain mostly in the outer regions of the host's disc ($R \geq 10\,\text{kpc}$), as the values of A_2/A_0 in the inner part ($R \leq 10\,\text{kpc}$) are almost zero (see Fig. 4.4). After $t \sim 3\,\text{Gyr}$ or so, the spiral starts to decay, as shown by the decreasing values of the A_2/A_0. By the end of the simulation run, the values of the A_2/A_0 become almost zero, implying that the tidally-induced spirals have wound up almost completely.

Next, we quantify the temporal evolution of the strength of the tidally-induced spirals in our fly-by model *RP40i00pro*. Following Sellwood & Carlberg (1984a), Sellwood & Athanassoula

Fig. 4.4 The evolution of the coefficient of the $m = 2$ Fourier moment (A_2/A_0) in the $R - t$ space for the model *RP40i00pro*. A logarithmic binning is employed along the radial direction, for details see text. The colour bar shows the values of A_2/A_0. The vertical dashed line (in cyan) denotes the time of the pericentre passage of the perturber.

(1986), Puerari et al. (2000), we define

$$A(m, p) = \frac{\sum_j^N m_j \exp[i(m\phi_j + p \ln R_j)]}{\sum_j^N m_j}, \tag{4.8}$$

where $|A(m, p)|$ is the amplitude, m_j is the mass of jth star, m is the spiral arm multiplicity, (R_j, ϕ_j) are the polar coordinates of the jth star in the plane of the disc, and N is the total number of stellar particles in the annulus $R_{min} \leq R \leq R_{max}$ within which the spiral feature exists and/or is most prominent. Here, we take $R_{min} = 3R_d$, and $R_{max} = 6R_d$, where $R_d = 3.8$ kpc. For this annular region, we estimate $A(m, p)$ as a function of p for $p \in [-50, 50]$, with a fixed step of $dp = 0.25$ (as suggested by Puerari et al., 2000) for different values of m. Then, we evaluate the parameter p_{max} which corresponds to the maximum value of $|A(m, p)|$. We find that the amplitude $|A(m, p = p_{max})|$ shows a maximum value for $m = 2$, indicating that the $m = 2$ spiral is the strongest. Therefore, at a certain time t, we define the amplitude $|A(m = 2, p = p_{max})|$ as the strength of spirals (Oh et al., 2015a; Semczuk et al., 2017; Kumar et al., 2021). The resulting temporal evolution of the strength of spirals for the model *RP40i00pro* is shown in Fig. 4.5. As seen from Fig. 4.5, the tidally-induced spirals grow for some time after the interaction happens, then remain stable for about 1 Gyr before weakening from around $t = 3$ Gyr. By the end of

Fig. 4.5 Temporal evolution of the spiral strength ($|A(m = 2, p = p_{\max})|$), calculated using Eq. 4.8 for the model *RP40i00pro*. The tidally-induced spirals decay by the end of the simulation run. The horizontal black dotted line denotes $|A(m = 2, p = p_{\max})| = 0.1$, used as an operational definition for the onset of the spirals, see text for details. The vertical dashed line (in cyan) denotes the time of the pericentre passage of the perturber.

the simulation, the spirals' strength becomes almost zero. For quantifying the longevity of the spirals, we define $|A(m = 2, p = p_{\max})| = 0.1$ as the onset of the strong spiral perturbation[2]. Therefore, the spirals persist for a time of $\sim 4.2\,$Gyr (after their formation) for the model *RP40i00pro* (also see Fig. 4.5).

4.3.2 Winding of the tidally-induced spirals

Lastly, we investigate the winding of the tidally-induced spirals in our fly-by model *RP40i00pro*. As shown in past *N*-body simulations of galactic discs (e.g. see Oh et al., 2008; Struck et al., 2011; Kumar et al., 2021), a spiral arm can wind up with time. Following Oh et al. (2015a), and Semczuk et al. (2017), at a certain time t, we define the pitch angle, α, as $\alpha = \tan^{-1}(m/p_{\max})$ using Eq. 4.8. The resulting temporal evolution of the pitch angle for the model *RP40i00pro* is shown in Fig. 4.6. As revealed in Fig. 4.6, the temporal evolution of the pitch angle displays two distinct phases, namely, the initial rapid winding phase where the pitch angle decreases sharply, and the subsequent slow winding phase where the pitch angle decreases less drastically.

---

[2]The threshold value of 0.1 is used purely as an operational definition for the onset of the spirals.

Fig. 4.6 Temporal evolution of the pitch angle (α) of the tidally-induced spirals is shown for the model *RP40i00pro*. A double-exponential profile (see Eq. 4.9) is fitted to model the temporal evolution. The blue dashed line denotes the initial rapid winding phase whereas the green dashed line denotes the subsequent slow winding phase, for details see text. The red solid line denotes the best-fit double-exponential profile. The vertical dashed line (in cyan) denotes the time of the pericentre passage of the perturber.

To model the temporal evolution of the pitch angle, we fit a double-exponential profile having the form

$$\alpha = \alpha_1 \exp\left[-\lambda_1 (t - t_0)\right] + \alpha_2 \exp\left[-\lambda_2 (t - t_0)\right], \tag{4.9}$$

where α_1, α_2, λ_1, and λ_2 are free parameters. Here, $t_0 = 1\,\mathrm{Gyr}$, and denotes the time of the spirals' formation. The fitting is performed via the SCIPY package CURVEFIT which uses the Levenberg-Marquardt algorithm. The resulting best-fit double exponential profile is shown in Fig. 4.6. We define the winding time-scale, τ_{wind}, as $\tau_{\mathrm{wind}} = \alpha/|\dot{\alpha}|$ where $|\dot{\alpha}|$ denotes the (absolute) rate of change of the pitch angle with time. For the initial rapid winding phase, $\tau_{\mathrm{wind}} \simeq 1/\lambda_1$ whereas for the subsequent slow winding phase $\tau_{\mathrm{wind}} \simeq 1/\lambda_2$ (see Eq. 4.9). We find the best-fit values as $\lambda_1 = 2.51 \pm 0.05\,\mathrm{Gyr}^{-1}$, and $\lambda_2 = 0.18 \pm 0.01\,\mathrm{Gyr}^{-1}$ which translate to a winding time-scale $\tau_{\mathrm{wind}} = 0.4\,\mathrm{Gyr}$ for the initial rapid winding phase, and a winding time-scale $\tau_{\mathrm{wind}} = 5.7\,\mathrm{Gyr}$ for the subsequent slow winding phase for the model *RP40i00pro*.

4.3.3 Nature of tidally-induced spirals

So far, we have demonstrated that a fly-by interaction with a perturber induces a strong spiral feature in the outer disc of the host galaxy. However, the question remains whether these spirals are density waves or material arm in nature. For a comprehensive review on the nature of the

Fig. 4.7 *Nature of spirals:* top and bottom panels of the first column show the surface density and residual density maps of *RP*40*i*00*pro* model at $t = 2$ Gyr over-plotted with two patches (in magenta and cyan dots) of stellar particles in fully developed spiral arms. Right column is the analogous to the left column, but for $t = 2.4$ Gyr. Both patches of stars, initially associated along the spirals, have subsequently sheared out by the differential rotation, and left the spirals.

spirals in disc galaxies, the reader is referred to Binney & Tremaine (2008b), and Dobbs & Baba (2014).

For the set of the *N*-body models we are using here, we do not have the age information of the stellar particles. This, in turn, restricts us from dividing stellar particles into different age-bins to trace the existence of spirals in different stellar population with different ages, as previously done in Ghosh et al. (2022a). Therefore, following Grand et al. (2012); D'Onghia et al. (2013), we test the nature of the tidally-induced spirals by following the stars which are located on spirals arms at a certain time. We chose a time, say $t = 2$ Gyr when the spirals are fully-developed for the model *RP*40*i*00*pro*, and select two small patches (shown in magenta and cyan) of stars along the arms. This is shown in Fig. 4.7 (top left-hand panel). Now, if the spirals are of material arm in nature, then the stars would not leave the spiral arm at a subsequent time. To check that, we follow the selected stars at a later time, $t = 2.4$ Gyr (see top right-hand panel of Fig. 4.7). As seen clearly from Fig. 4.7, the stars, initially concentrated in small patches along the spiral arm, have sheared out due to the underlying differential rotation. This shearing out of the stars is more prominent in the distribution shown in the (R, ϕ)-plane (see bottom panels of Fig. 4.7). Interestingly, the stars, initially contained in the patches have left the spirals at a subsequent time, and the pitch angle of the selected stars is different from

Fig. 4.8 Temporal evolution of the spiral strength ($|A(m = 2, p = p_{\max})|$), calculated using Eq. 4.8 is shown for models with different angle of interaction, and orbital spin vector. For comparison, we kept the model $RP40i00pro$ here (see red solid line). For models with prograde configuration, the strength of spirals decreases monotonically from the co-planar ($i = 0°$) to polar ($i = 90°$) orbital configuration. The horizontal black dotted line denotes $|A(m = 2, p = p_{\max})| = 0.1$, used as an operational definition for the onset of the spirals.

that of the spirals at $t = 2.4$ Gyr. Furthermore, we calculate the pattern speed of the spirals at different radial locations in the disc. We find that the pattern speeds of the spirals are very different from the values of the circular frequency (Ω), calculated at different radial locations. For brevity, these are not shown here. This shows that the spirals present in our fly-by model $RP40i00pro$ are density waves in nature.

4.3.4 Dependence on orbital parameters

Here, we explore the generation of the spirals in the host galaxy due to a tidal interaction with the perturber galaxy for different angles of interaction of the tidal encounter as well as for different orbital spin vectors (prograde or retrograde). Intuitively, the response of the host galaxy will be different for polar ($i = 90°$) and co-planar ($i = 0°$) orbital configurations. First we consider the other prograde models, with angles of interaction $30°$ and $60°$ (for details see section 4.2.2). Both the models exhibit a similar trend of excitation of tidally-induced spirals in the disc of the host galaxy, as in model $RP40i00pro$. Shortly after the tidal encounter happens, the disc of the host develops a spiral feature which grows for a certain time, and then

starts decaying. We calculate the strength of the spirals using Eq. 4.8 at different times for these models. The resulting temporal variations of the strength of the spirals for these three models are shown in Fig. 4.8. The maximum strength of the tidally-induced spirals decreases monotonically with larger angle of interaction. Next, we use $|A(m = 2, p = p_{\max})| = 0.1$ for defining the onset of the strong spiral perturbation. We find that the spirals persist for a time-scale of $\sim 2.9 - 4.2\,\mathrm{Gyr}$, depending on the angle of interaction (also see Fig. 4.8). For the polar ($i = 90°$) configuration, the spirals are very weak as the values of $|A(m = 2, p = p_{\max})|$ remain below 0.1 (see Fig. 4.8).

Next, we study the strength of the spirals in a galaxy fly-by model where the perturber interacts with the host galaxy in a retrograde orientation (with $i = 0°$) (for details see section 4.2.2). A visual inspection of the face-on distribution of the stars (in the host galaxy) does not reveal any prominent spirals. We again calculate the strength of the spirals using Eq. 4.8 for the model *RP*40*i*00*ret*, and show this also in Fig. 4.8. The value of the amplitude $|A(m = 2, p = p_{\max})|$ remains close to zero throughout the evolution of the model *RP*40*i*00*ret*, indicating that no prominent spirals are triggered/excited by the tidal encounter with the perturber galaxy for this model.

Furthermore, we have checked that the tidally-induced spirals display a similar winding, namely, an initial rapid winding phase, followed by a slow winding phase, as seen for the model *RP*40*i*00*pro* (see Fig. 4.6). Also, we have checked the nature of the resulting spirals for these other models, using the same technique employed in section 4.3.3. We find that for the models showing prominent spirals, the resulting spirals show a density wave nature, similar to the model *RP*40*i*00*pro*. For the sake of brevity, we do not show these results here.

4.4 Breathing motions excited by tidally-induced spirals

In the previous section, we have demonstrated that a tidal interaction with a perturber galaxy can excite prominent spirals in the disc of the host galaxy. Here, we investigate the dynamical impact of these tidally-induced spirals on the bulk vertical motions of the host galaxy. We first choose the fly-by model *RP*40*i*00*pro* which harbours a strong spiral after the interaction. We choose the radial extent $15 \leq R/\,\mathrm{kpc} \leq 17$ where the spirals are prominent at later times in this radial annulus, and we calculate the mean vertical velocity ($\langle v_z \rangle$) in the (ϕ, z)-plane at four different times, namely, at $t = 1, 2, 4$, and $6\,\mathrm{Gyr}$ (same as in Fig. 4.2). This is shown is Fig. 4.9. During our chosen time interval, spirals' strength varies from strong to weak (for details see section 4.3.1). At $t = 1\,\mathrm{Gyr}$, just after the fly-by encounter, the distribution of the bulk vertical velocity ($\langle v_z \rangle$) in the (ϕ, z)-plane predominantly shows bending motions, i.e., stellar particles on both sides of the mid-plane are moving coherently in the same direction. At this time, a

Fig. 4.9 Distribution of the mean vertical velocity, $\langle v_z \rangle$, in the (ϕ, z)-plane, for stars in the radial annulus $15 \leq R/\,\mathrm{kpc} \leq 17$ are shown at four different times for the model $RP40i00pro$. The solid black lines denote the contours of constant density. The presence of large-scale, non-zero vertical velocities for stellar particles are seen at all four time-steps, for details see text.

Fig. 4.10 Distribution of the breathing velocity, V_{breath} (Eq. 4.10) at different vertical distances from the mid-plane, at four different times for the model *RP40i00pro*. The solid black lines denote the contours of constant density. The *top panels* show stars at $|z| = [0, 400]$ pc whereas the *bottom panels* show stars at $|z| = [400, 1200]$ pc, respectively.

prominent spiral is yet to form in the host galaxy (see Fig. 4.4). However, by $t = 2$ Gyr, the host galaxy shows a prominent spiral (see Fig. 4.4), and the distribution of the bulk vertical velocity ($\langle v_z \rangle$) in the (ϕ, z)-plane changes drastically. Now, the stellar particles on both sides of the mid-plane are moving coherently towards or away from it, indicating vertical breathing motion dominates. The relative sense of the $\langle v_z \rangle$ varies as a function of the azimuthal angle. However, at $t = 4$ Gyr when the tidally-induced spirals have weakened substantially, the distribution of the $\langle v_z \rangle$ is again seen to be dominated by the bending motions. By the end of the simulation ($t = 6$ Gyr), the spiral has wound up, and the distribution of $\langle v_z \rangle$ remains dominated by the bending motions of the stars.

To quantify the breathing motions, we define the breathing velocity, V_{breath}, as (Debattista, 2014; Gaia Collaboration et al., 2018; Ghosh et al., 2022a)

$$V_{breath}(x, y) = \frac{1}{2} \left[\langle v_z(x, y, \Delta z) \rangle - \langle v_z(x, y, -\Delta z) \rangle \right], \qquad (4.10)$$

where $\langle v_z(x, y, \Delta z) \rangle$ is the mean vertical velocity at position (x, y) in the galactocentric cartesian coordinate system, averaged over a vertical layer of thickness Δz (for details see Gaia Collaboration et al., 2018; Ghosh et al., 2022a). A positive breathing velocity ($V_{breath} > 0$) implies that the stars are coherently moving away from the mid-plane (expanding breathing motion), while

$V_{\mathrm{breath}} < 0$ implies that the stars are moving coherently towards the mid-plane (compressing breathing motion). We find that at larger heights, the particle resolution of our selected fly-by model is not well-suited to compute meaningful values of V_{breath}. Therefore, we calculate the distribution of V_{breath} in the (x, y)-plane for two vertical layers, namely, $|z| = [0, 400]$ pc and $|z| = [400, 1200]$ pc. The resulting distributions of V_{breath} at the same four times (as in Fig. 4.9) are shown in Fig. 4.10. The breathing velocity is close to zero near the mid-plane, for all four times considered. However, at $t = 2$ Gyr when the strong spirals are present in the host galaxy, the upper vertical layer ($|z| = [400, 1200]$ pc) shows significant, coherent breathing velocity ($\sim 6 - 8 \, \mathrm{km\,s^{-1}}$, in magnitude). This trend is similar to what is shown for spiral-driven breathing motions (e.g., see Debattista, 2014; Ghosh et al., 2022a), and is also similar to the breathing motions seen from the *Gaia* DR2 (Gaia Collaboration et al., 2018). A visual inspection also reveals that at $t = 2$ Gyr, the compressing breathing motions are closely associated with the spiral arms whereas the expanding breathing motions arise in the inter-arm regions (see Fig. 4.10), similar to the findings of Faure et al. (2014), Debattista (2014) and Ghosh et al. (2022a). However, after $t = 4$ Gyr when the spirals are either significantly weaker or completely absent, the host galaxy does not show any prominent, coherent breathing motion in the upper layer ($|z| = [400, 1200]$ pc). To probe this further, we calculate the distribution of V_{breath} at two vertical layers in every 200 Myr from $t = 2$ Gyr to $t = 6$ Gyr for the model *RP40i00pro*. For the sake of brevity, this is not shown here. We find that the incidence of a prominent, coherent breathing motion is strongly related with the presence of a strong spiral feature in the host galaxy. This, together with the amplitude of the breathing velocity increasing with height (a signature of spiral-driven breathing motion, see Ghosh et al., 2022a) indicate that the breathing motions present in the host galaxy are driven by the tidally-induced spirals.

Finally, we study the azimuthal variation of the breathing motions and their connection with the peak(s) of the spirals. Figs. 4.9 and 4.10 already demonstrated that the breathing motions in model *RP40i00pro*, is associated with the spiral arm and the inter-arm regions. Here, we explore this further. Following Ghosh et al. (2022a), we quantify these breathing motions whose amplitude increases with height from the mid-plane, by fitting a straight line. In this formalism, the presence of a prominent breathing motion would result in a significantly *non-zero* slope. Furthermore, an expanding (positive) breathing motion will yield a positive slope whereas a compressing (negative) breathing motion will yield a negative slope. On the other hand, the non-zero value of the intercept of the best-fit straight line indicates the presence of a bending motion. For details, the reader is referred to Ghosh et al. (2022a). A similar approach was also used in Widrow et al. (2014). We consider the same radial extent $15 \, \mathrm{kpc} \leq R \leq 17 \, \mathrm{kpc}$ where a prominent spiral is present at $t = 2$ Gyr. Since the phase-angle of the $m = 2$ Fourier mode (ϕ_2) varies as a function of radius, indicating that the azimuthal

Fig. 4.11 Variation of the slopes as a function of the rotated azimuthal angle (ϕ') at four different times (having different spiral strength) for the model *RP40i00pro* (see blue dashed lines). Only the stellar particles in the radial extent $15 \leq R/\,\mathrm{kpc} \leq 17$ are chosen here. The black dashed line denotes the residual surface density ($\tilde{\Sigma}(R, \phi')$) as a function of the azimuthal angle, calculated in this chosen radial extent. The particles have first been binned in 1 kpc annuli and then azimuthally rotated relative to each other so that the minimum in $\tilde{\Sigma}(R, \phi))$ in each annulus is coincident, and then the mean vertical velocity ($\langle v_z \rangle$) is calculated, amounting to stacking different radial ranges while unwinding the spiral. For details, see text. The sense of rotation is towards increasing ϕ'.

locations of the density peaks vary as a function of radius, to obtain a stronger signal of the slope, we first rotate the stellar particles in two different $1\,$kpc-wide radial bins, in such a way that the density peaks in our chosen radial extent coincide. Then, we recalculate the slope of the breathing velocity as a function of the rotated azimuthal angle (ϕ'). The resulting variation of the slope as well as the residual surface density ($\tilde{\Sigma}(R, \phi')$) with rotated azimuthal angle (ϕ') are shown in Fig. 4.11 for four different times. As seen clearly from Fig. 4.11, at $t = 2\,$Gyr when the spirals are quite strong ($|A(m = 2, p = p_{\mathrm{max}})| = 0.38$), the compressive breathing motions are associated with the peak(s) of the spiral whereas the expanding breathing motions are associated with the density minima. This trend is consistent with the signature of a spiral-driven breathing motions as shown in Debattista (2014) and Ghosh et al. (2022a). We show that in our model, the amplitudes of the expanding breathing motions are higher than that of the compressing breathing motions – a trend attributed to the fact of having a more abrupt density variation as the stellar particles leave the spirals compared with when they enter the spirals (see Debattista, 2014). At $t = 3.5\,$Gyr, when the spirals are decaying ($|A(m = 2, p = p_{\mathrm{max}})| = 0.21$), the corresponding (absolute) values of the slope also decrease, thereby indicating that the breathing motions also weaken. By $t = 6\,$Gyr, the spirals get wound up (almost) completely ($|A(m = 2, p = p_{\mathrm{max}})| = 0.04$), and the (absolute) values of the slope are close to zero, indicating the absence of breathing motion. This further strengthens the case for the breathing motions being driven by the spirals, and not the *direct* dynamical consequence of a fly-by interaction. The breathing motions are seen to last for $\sim 1.5 - 2\,$Gyr after their generation for the model *RP*40*i*00*pro*.

We repeated this whole set of analyses for all the other models, to investigate the breathing motions and their connection with the incidence and strength of spirals present in the model. We find that a strong spiral always drives a prominent breathing motion with expanding breathing motions associated with the inter-arm region whereas the compressing breathing motions are associated with the peak(s) of the spirals. As the spirals get wound up, the breathing motions also cease to exist in the fly-by models. These trends are similar to what we have found for the model *RP*40*i*00*pro*. Therefore, for brevity, these are not shown here. In the fly-by model with retrograde orbital configuration (*RP*40*i*00*ret*), the spiral structure itself is weaker compared to the other models in prograde orbital configuration (see Fig. 4.8). We checked that no prominent breathing motion is excited by this feeble spiral structure.

4.5   Discussion

Our fly-by interactions excite strong spirals in the outer regions of the host galaxy's disc. The spirals show a variation in their maximum strength depending on the angle of interaction, and

the orbital spin vector. Here, we compare the strength, location, and nature of the tidally-induced spirals in our models with past studies. The numerical simulations of Oh et al. (2015b) showed that in their models, a stronger tidal encounter induces prominent spirals in the inner regions ($5 \leq R/\,\text{kpc} \leq 10$) of the host galaxy's disc (in addition to the tidal tails in the outer parts). The arm strength of the spirals vary in the range $\sim 0.1 - 0.18$, depending on the values of the relative tidal force, and the relative imparted momentum. Moreover, the spirals are of kinematic density wave in nature. Also, the simulations of Semczuk et al. (2017) showed the generation of transient spirals due to the tidal force exerted by the potential of a cluster; the spirals appear with each pericentre passage, followed by a fast decay. These spirals appear in the outer disc region ($12 \leq R/\,\text{kpc} \leq 17$), with the maximum arm strength varying in the range $\sim 0.4 - 0.75$. In comparison, the spirals in our fly-by models are most prominent in the outer regions ($R \geq 10\,\text{kpc}$). The maximum arm strength varies in the range $\sim 0.15 - 0.38$, depending on the angle of interaction, and the orbital spin vector (see Fig. 4.8). The spirals appear shortly after the pericentre passage of the perturber, grow rapidly, followed by winding up of those spirals. This winding of spirals show two distinct phases, namely, the initial rapid winding phase, followed by a slow winding phase (see Fig. 4.6).

The amplitude of the spiral-driven breathing motions also merits some discussion. In our fly-by models, when the spirals are most prominent, the values of the best-fit slope (used as a proxy for the breathing amplitude) vary from $\sim -5\,\text{km}\,\text{s}^{-1}\,\text{kpc}^{-1}$ to $\sim 10\,\text{km}\,\text{s}^{-1}\,\text{kpc}^{-1}$ (see Fig. 4.11). In comparison, the fly-by model (with spirals present) of Widrow et al. (2014) showed the (absolute) values of the best-fit slope $\sim 10\,\text{km}\,\text{s}^{-1}\,\text{kpc}^{-1}$. As for the spiral-driven breathing motions where the spirals arise due to internal instability, Ghosh et al. (2022a) reported the values of the slope varying from $\sim -2.5\,\text{km}\,\text{s}^{-1}\,\text{kpc}^{-1}$ to $\sim 3\,\text{km}\,\text{s}^{-1}\,\text{kpc}^{-1}$. Furthermore, the strong spirals present in the models used by Faure et al. (2014); Debattista (2014) can drive large breathing motions ($|\langle v_z \rangle| \sim 5 - 20\,\text{km}\,\text{s}^{-1}$).

We have considered only *N*-body models of an unbound, fly-by interaction, excluding the interstellar gas. It is well known that a disc galaxy contains a finite amount of gas (e.g., see Scoville & Sanders, 1987). Additionally, in the ΛCDM galaxy formation scenario, a galaxy can accrete cold gas (e.g., Birnboim & Dekel, 2003; Kereš et al., 2005; Dekel & Birnboim, 2006; Ocvirk et al., 2008) either during the merger-phase or at a later stage. Past studies have shown the dynamical importance of the interstellar gas in the context of cooling the stellar disc and facilitating the generation of fresh spiral waves (Sellwood & Carlberg, 1984b), and in maintenance of spiral density waves in infinitesimally-thin discs (Ghosh & Jog, 2015, 2016) as well as in a galactic disc with finite thickness (Ghosh & Jog, 2022). For gas rich galaxies undergoing such fly-by interactions, the vertical breathing motion may be important for increasing the turbulence in the gas where star formation is insignificant (Stilp et al., 2013).

This is because as the stars are in the breathing motion, the vertical potential will change with time and the gas distribution will be affected. However, since the vertical stellar velocity induced by the breathing motion is fairly small (of the order of a few km s^{-1}), this effect may not be very significant, especially if compared to the much larger kinematic effect of supernova explosions and stellar winds (Krumholz et al., 2018; Yu et al., 2021). In addition, galaxy bulges play an important role in maintaining spiral density waves in the disc for a longer time (Saha & Elmegreen, 2016). Although, our galaxy models have a classical bulge, we have not varied the contribution of the bulges in our models.

Lastly, our models are specifically designed in a way that the unperturbed disc galaxy is not bar unstable and only forms weak spirals; the strong spirals that form therefore are a dynamical result of the fly-by encounter. However, in reality, the Milky Way also harbours a stellar bar (e.g., see Liszt & Burton, 1980; Binney et al., 1991; Weinberg, 1992; Binney et al., 1997; Blitz & Spergel, 1991; Weiland et al., 1994; Dwek et al., 1995; Freudenreich, 1998; Hammersley et al., 2000; Wegg & Gerhard, 2013). Furthermore, the bulk vertical motions in the Solar neighbourhood and beyond, display both bending and breathing motions (e.g., Gaia Collaboration et al., 2018; Carrillo et al., 2018). This simultaneous presence of bending and breathing motions could well be collectively manifested due to a combination of internal (spiral and/or bar-driven) and external driving mechanisms (tidal encounters), as previously investigated by Carrillo et al. (2019). We stress that the aim of this work is to clarify whether the excitation of breathing motions are 'directly' related to tidal interactions or whether they are driven by the tidally-induced spirals (as also mentioned in section 7.1), and not to replicate the observed dynamical state of the Milky Way.

4.6   Summary

In summary, we investigated the dynamical impact of an unbound, single fly-by interaction with a perturber galaxy on the generation of the tidally-induced spiral features and the associated excitation of vertical breathing motions. We constructed a set of N-body models of fly-by encounter, with mass ratio kept fixed to 5:1 while varying different orbital parameters. Our main findings are :

- Fly-by interactions trigger a strong spiral structure in the disc of the host galaxy. The spirals grow rapidly in the initial times, followed by a slow decay. The generation and the strength of these tidally-induced spirals depend strongly on the angle of interaction as well as on the orbital spin vector. For the same orbital energy and the angle of interaction,

the models in prograde configuration are more efficient at driving strong spirals when compared to models in retrograde configuration.

- The tidally-induced spirals in the host galaxy can survive for $\sim 2.9 - 4.2\,\text{Gyr}$ after their formation. The pitch angle of the resulting spirals display two distinct phases of winding, namely, a fast winding phase ($\tau_{\text{wind}} \sim 0.4\,\text{Gyr}$) and a subsequent slow winding phase ($\tau_{\text{wind}} \sim 5.7\,\text{Gyr}$).

- When the tidally-induced spirals are strong, they drive coherent, large-scale vertical breathing motions whose amplitude increases with height from the mid-plane. Furthermore, the azimuthal locations of the compressing breathing motions ($V_{\text{breath}} < 0$) are associated with the peaks of the spirals whereas the azimuthal locations of the expanding breathing motions ($V_{\text{breath}} > 0$) coincide with the density minima of the spirals. These trends are in agreement with the signatures of spiral-driven breathing motions.

- The temporal evolution of these breathing motions follow closely the temporal evolution of the strengths of the spirals. A stronger spiral drives breathing motions with larger amplitudes. These breathing motions, excited by tidally-induced spirals, can persist for $\sim 1.5 - 2\,\text{Gyr}$ in the disc of the host galaxy.

Thus, the results presented in this chapter demonstrate that a strong spiral structure can drive large, coherent vertical breathing motions irrespective of their formation scenario, i.e., whether induced by tidal interactions (as shown here) or generated via internal disc gravitational instability (e.g., Faure et al., 2014; Debattista, 2014; Ghosh et al., 2022a). Furthermore, our results highlight the cautionary fact that although in past studies, the tidal interactions are considered as the 'usual suspect' for driving the vertical breathing motions, it is indeed the tidally-induced spirals which drive the breathing motions, and the dynamical role of such tidal encounters remains only ancillary.

4.7   Appendix: Evolution in isolation

So far, we have shown that in our fly-models, a prominent spiral appears shortly after the interaction happens, and this spiral drives a coherent vertical breathing motion in the host galaxy. However, it remains to be investigated whether the host galaxy, when evolved in isolation, could still generate spirals and the associated vertical breathing motions. We evolve the host galaxy model in isolation, for $6\,\text{Gyr}$. A visual inspection of the face-on density distribution of the stellar particles reveals no prominent spirals, throughout the simulation run, (as can be seen in the density contours in Fig. 4.13). Following the methodology described in

Fig. 4.12 Temporal evolution of the spiral strength ($|A(m = 2, p = p_{\mathrm{max}})|$), calculated using Eq. 4.8 is shown for the isolated host galaxy model. For comparison, we kept the model *RP40i00pro* here (see red solid line). The isolated model does not show any prominent spirals throughout the simulation run.

Section 4.3.1, we measure the values of $|A(m = 2, p = p_{\mathrm{max}})|$ (used to quantify the strength of spirals) for the isolated host galaxy model. The resulting temporal evolution is shown in Fig. 4.12. The values of $|A(m = 2, p = p_{\mathrm{max}})|$ remain close to zero throughout the simulation run, demonstrating that no prominent spiral arm is generated during the entire isolated evolution of the host galaxy model.

Furthermore, we calculate the breathing motions (V_{breath}), using Eq. 4.10, for both the vertical slices, namely, $|z| = [0, 400]\,\mathrm{pc}$ and $|z| = [400, 1200]\,\mathrm{pc}$. As before, the vertical slice $|z| = [0, 400]\,\mathrm{pc}$ does not show any breathing motion. Interestingly, the upper vertical slice ($|z| = [400, 1200]\,\mathrm{pc}$) does not show any prominent breathing motion either throughout the simulation run (see Fig. 4.13), in sharp contrast with the fly-by models (compare with Fig. 4.10). This clearly demonstrates that the spirals in the fly-by models are indeed tidally-induced. In other words, the generation of spirals and the associated spiral-driven vertical breathing motions can indeed be attributed to the dynamical impact of a fly-by interaction.

Fig. 4.13 Evolution in isolation: distribution of the breathing velocity, V_{breath}, for stars at $|z| = [400, 1200]$ pc at four different times for the isolated host galaxy model. The solid black lines denote the contours of constant density. No prominent vertical breathing motion is seen for the isolated evolution, in sharp contrast with the fly-by models (e.g., compare with Fig. 4.10).

5

The Effect of Dark Matter Halo Shape on Bar Buckling and Boxy/Peanut Bulges

5.1   Introduction

The fraction of bars in the observable Universe varies from 30% to 70% (Aguerri et al., 2009; Masters et al., 2011; Díaz-García et al., 2016). Bars are the major driver of secular evolution in disc galaxies (Weinberg & Katz, 2007; Athanassoula, 2013; Long et al., 2014; Gadotti et al., 2020) and play a crucial role in the re-distribution of the mass and angular momentum among different components of a galaxy (Athanassoula, 2002, 2003; Saha et al., 2012b, 2016b; Kataria & Das, 2019). Studies have shown that during disc evolution bars themselves also evolve, both in length and vertical thickening. The latter can result in the formation of boxy/peanut/x-shape (in short BPX) pseudo-bulges (Friedli & Pfenniger, 1990; Debattista et al., 2006; Gadotti, 2011).

The origin of the vertical thickening of bars has been studied widely using numerical simulations (Combes & Sanders, 1981; Combes et al., 1990a; Raha et al., 1991b; Bureau & Athanassoula, 2005). There are mainly three mechanisms which lead to bar thickening:- (i) bar buckling (Combes et al., 1990a; Raha et al., 1991b), (ii) the 2:1 vertical resonance (Quillen

et al., 2014), and (iii) the gradual trapping of stellar orbits into the 2:1 resonance (Sellwood & Gerhard, 2020). Bar buckling is the most violent thickening mechanism during which the bar bends out of the disc plane. The buckling reduces the size and strength of the bar and results in the formation of boxy/peanut bulges (Sellwood & Merritt, 1994; Debattista et al., 2005; Martinez-Valpuesta & Shlosman, 2004). In some cases where buckling is slow and less energetic, it can even destroy the bar (Collier, 2020). There are evidences of multiple (recurrent) buckling in numerical simulations (Martinez-Valpuesta et al., 2006b; Łokas, 2019b). The primary buckling occurs in the inner part of the bar and persists for less than 1 Gyr; however the secondary buckling take place in the outer region of the bar and remains for few *Gyr* (Martinez-Valpuesta et al., 2006b; Athanassoula, 2016a). In this chapter we focus only on bar buckling, which is the most rapid process that produces the vertical thickening of bars.

Some studies suggest that properties of disc, bulge, and gas affect bar buckling. For example in their N-body simulations, Friedli & Pfenniger (1990) found that a small asymmetry about the mid plane of the disc accelerates bar buckling. The presence of a classical bulge in the center of galaxies may also prevent the onset of buckling instability in bars (Smirnov & Sotnikova, 2019). The warm gas in the galaxies has also been found to decrease the distortion in a bar which is due to buckling (Łokas, 2020). Buckling time and strength remain unaltered in galaxy flybys (Kumar et al., 2021).

On a large scale, bar formation and evolution are affected by dark matter halo properties. The presence of live dark matter halo supports the bar formation instability, in contrast to rigid dark matter halo that delay onset of bar formation (Athanassoula, 2002; Saha & Naab, 2013). There have been several studies of angular momentum re-distribution between bar and live dark matter halo (Sellwood, 1980; Athanassoula, 2003; Martinez-Valpuesta et al., 2006b; Collier et al., 2019). The detailed study of the effect of halo triaxiality and gas mass fraction on the formation and evolution of the bars is discussed in Berentzen et al. (2006) and Athanassoula et al. (2013). Recently, Collier et al. (2018) have studied the evolution of bars in rotating and non-spherical live halos. They noticed multiple (=two) buckling events only in prolate halo. The detailed nature of bar buckling in non-spherical halos and the evolution of the buckling induced boxy/peanut bulges are not well explored.

There have been a few attempts to find the signatures of ongoing buckling in the observable Universe. However, the time period of bar buckling is very short and the presence of a central concentration can halt the buckling instability, so it is not easy to detect ongoing bar buckling in galaxies. The first attempt used the bar isophotes and the kinematic signatures of bar buckling derived from simulations to detect buckling events in NGC 3227 and NGC 4569 (Erwin & Debattista, 2016). Recently, Xiang et al. (2021) have used kinematic signatures to detect ongoing buckling in face-on galaxies.

In this chapter we show that the non-spherical nature of halos affects bar buckling significantly and has very important implications for the observations of bars. We vary the halo shape from oblate to prolate, keeping the ratio of halo axes equal in the disc plane, which is a fairly good assumption as shown in recent numerical studies (Bett et al., 2010; Liao et al., 2017). We have also characterized the properties of the buckling induced boxy/peanut bulges in oblate, spherical and prolate halos.

5.2   Simulations And Analysis

We have simulated isolated disc galaxies with different dark matter halo shapes ranging from oblate to prolate including spherical. All of our model galaxies form a bar and undergo buckling instability. As a result of buckling, the bar thickens and forms a boxy/peanut bulge. To trace the properties of the buckling induced boxy/peanut pseudo-bulge, we have evolved all the models until 8 Gyr. The model setup has been described in detail in earlier papers (Kataria & Das, 2018; Kumar et al., 2021), and so we describe it only briefly below.

5.2.1   Model Galaxies:

We used a publicly available open source code GALIC (Yurin & Springel, 2014) to generate the initial conditions of our model galaxies. GALIC populates the particles according to the given density distribution and finds the equilibrium solution of the collisionless Boltzmann Equation (CBE) by iteratively changing the initial velocities of the particles. This code makes use of Schwarzschild's method and made-to-measure technique (see Yurin & Springel (2014)).

Each of our model galaxies incorporates a stellar disc and a dark matter halo. Since we are interested in bar buckling and the resulting pseudo-bulges, we did not include the bulge component in our models because the presence of a spherical potential in the center of a galaxy slows down bar formation and hence hinders the buckling of bars (Kataria & Das, 2018). To populate the particles of a spherical dark matter halo, we used the Hernquist density profile (Hernquist, 1990) defined as,

$$\rho_{dm}(r) = \frac{M_{dm}}{2\pi} \frac{a}{r(r+a)^3} \tag{5.1}$$

where M_{dm} is the total mass of the dark matter halo and 'a' is its scale radius and is related to the concentration parameter 'c' of the NFW halo (Navarro et al., 1996) with mass $M_{200} = M_{dm}$

Table 5.1 Initial parameters of the model galaxies.

Total mass (M)	$6.4 \times 10^{11} M_\odot$
Halo spin parameter (λ)	0.035
Halo concentration parameter (c)	20
Disc mass fraction	0.10
Disc scale radius (R_s)	2.90 kpc
Disc scale height (z_0)	0.58 kpc
Halo particles (N_{Halo})	2.0×10^6
Disc particles (N_{Disc})	2.0×10^6
Total particles (N_{Total})	4.0×10^6

by the relation,

$$a = \frac{r_{200}}{c} \sqrt{2 \left[\ln(1+c) - \frac{c}{(1+c)} \right]}, \tag{5.2}$$

where r_{200} is the radius of the NFW halo. It is defined as the radius of the sphere within which the average density is 200 times the critical density of the Universe and M_{200} is the mass within this radius.

The non-spherical halos (oblate and prolate) are generated by linearly distorting the spherical halo along the z-axis perpendicular to the disc plane. Assuming an ellipsoid has $a_x = b_y$, and c_z as its three axes, then the ratio $q = c_z/a_x$ defines the shape of the halo. An oblate halo has $q < 1$, a spherical halo has $q = 1$, and a prolate halo has $q > 1$. The density profile of such non-spherical halos is given by

$$\bar{\rho}_{dm}(R,z,q) = \frac{1}{q} \rho_{dm} \left(\sqrt{R^2 + \frac{z^2}{q^2}} \right) \tag{5.3}$$

where ρ_{dm} is the Hernquist density profile as shown in equation (6.2). This new profiles keeps the total mass of the halo invariant.

The distribution of particles in the stellar disc is represented by the exponential profile in the radial direction and the sech^2 profile in the direction perpendicular to the disc. So the net density profile is given as,

$$\rho_d(R,z) = \frac{M_d}{4\pi z_0 R_s^2} \exp\left(-\frac{R}{R_s} \right) \text{sech}^2 \left(\frac{z}{z_0} \right), \tag{5.4}$$

where M_d is the total disc mass, z_0 is the disc scale height and R_s is the disc scale radius.

We have simulated five models of non-spherical dark matter halos in isolated disc galaxies for the halo shape parameter $q \in \{0.70, 0.85, 1.00, 1.15, 1.30\}$. Each of our model galaxies has

Fig. 5.1 Initial condition of model galaxies. Panels from left to right show the rotation curve, Toomre instability parameter, and vertical velocity dispersion of model disc galaxies respectively. Different colors in the legend represent the models with different the halo shape parameters.

2×10^6 dark matter particles and 2×10^6 stellar particles making a total of 4×10^6 particles. For testing purposes, we have also simulated all the models with 2×10^6 total particles and found similar results as that for 4×10^6 total particles. The total mass of each galaxy is set to $6.4 \times 10^{11} M_\odot$, where each galaxy contains 90% dark matter mass and 10% stellar mass. Table 5.1 summarises the initial parameters of our model galaxies which are standard in all the models. The rotation curve $v(R)$, Toomre Q parameter, and the vertical velocity dispersion σ_z are shown in Fig. 5.1. Since the equilibrium model of the galaxy is generated by iteratively solving CBE for disc and halo, the rotation velocity, Toomre Q, and the velocity dispersion of oblate halos are always a little higher than the respective prolate halos.

After generating initial realizations of model galaxies, we evolved them using the open source code Gadget-2 (Springel et al., 2001a; Springel, 2005a,b) upto 8 Gyr. Gadget-2 is a massively parallelized and adaptive (in both space and in time) code. The gravitational softening for stellar particles and dark matter particles are set to 0.02 kpc and 0.03 kpc respectively. The maximum percentage change in the total angular momentum of the galaxy is well within 0.1% for all model galaxies throughout the evolution.

We used the shape parameter $'q'$ of the halos for naming the models. For example 'q1.15' represents the initial galaxy model with shape parameter $q = 1.15$. However, the shape of the halo may change after evolution of the models. It should be noted that all the quantities discussed here will be in units of the dimensionless Hubble parameter $'h'$ where the Hubble constant is defined as $H_0 = 100 \, h \, km \, s^{-1} Mpc^{-1}$.

5.2.2 Analysis:

The bar strength is usually determined using the amplitude of the m=2 Fourier mode relative to m=0 Fourier mode. The amplitude of the mth Fourier mode at a cylindrical radius R is given by,

$$A_m(R) = \left| \sum_{j=1}^{N} m_j \exp\left(im\phi_j\right) \right|, \tag{5.5}$$

where m_j is the mass and ϕ_j is the azimuth angle of the jth particle at a radius R, and N is the total number of particles at a radius R. The strength of the bar is defined as $A_{bar} = max\left[\frac{A_2}{A_0}(R)\right]$.

For the quantification of the buckling instability, we have adopted the commonly used expressions in the literature (Debattista et al., 2006; Xiang et al., 2021), i.e. the $m = 2$ Fourier mode weighted by the vertical velocity

$$A_{buck,v_z}(R) = \left| \frac{\sum_{j=1}^{N} v_{z_j} m_j \exp\left(2i\phi_j\right)}{\sum_{j=1}^{N} m_j} \right|, \tag{5.6}$$

and the $m = 2$ Fourier mode weighted by the vertical height

$$A_{buck,z}(R) = \left| \frac{\sum_{j=1}^{N} z_j m_j \exp\left(2i\phi_j\right)}{\sum_{j=1}^{N} m_j} \right|. \tag{5.7}$$

Equation 5.6 is a very useful relation for quantifying buckling in face-on galaxies where stellar kinematic is known while equation 5.7 quantifies the buckling in edge-on galaxies where vertical stellar morphology is known. We have also used median vertical height (z_{med}) for the quantification of buckling. The evolution of the boxy/peanut/x-shape structure is traced using the root mean square vertical height ($A_{BPX} = z_{rms}$). For all these calculations, we have considered only those stellar particles which lie above and below the mid plane within 10 disc scale height ($10z_0$). This choice reduces the discreteness noise from the measurement when some particles reach higher vertical distances.

5.3   Results

We have evolved isolated disc galaxy models with varying dark matter halo shapes ranging from q=0.70 (oblate) to q=1.30 (prolate). All the models are bar unstable and show bar buckling instability after reaching maximum bar strength. In the following subsections, we discuss how bar buckling is affected by halos shape and how it affects the final product, the boxy/peanut pseudo-bulge.

Fig. 5.2 Evolution of bar strength in non-spherical dark matter halos with different shape parameters $'q'$. The positions of bar formation and re-growth after buckling are marked with '+' symbol.

5.3.1   Dependence of bar formation time and bar strength on halo oblateness:

In Fig. 5.2, we have shown the time evolution of the bar strength in our model galaxies. We have smoothed the bar strength with the Savitzky–Golay filter (Savitzky & Golay, 1964) using window size = 5, and polynomial order = 3 for the filter parameters. The main motive for using this filter was to remove the noise and reveal the global evolution of the bar instability. All the models show a quick rise in bar strength. After reaching maximum strength, the bar strength decreases a little and again starts increasing. Later in this section we will show that this drop in bar strength is the result of bar buckling (or bending) event which weakens the bar. Finally, the strength of the bar saturates in all the models.

The effect of non-spherical dark matter halos on the bar formation is clearly visible. Prolate halos promote early bar formation, whereas oblate halos delay bar formation (where we consider the bar formation time to be the time from the beginning of simulation to the peak bar strength just before the decrease due to bar buckling). If we consider the galaxy with a spherical halo as the control model, oblate halos delay bar formation by more amount of time compared to the corresponding time taken for prolate halos to promote early bar formation.

At the time of bar formation, the amplitude of bar strength is always higher in non-spherical halos. The decrease in bar amplitude due to buckling is highest for prolate halos and lowest

for oblate halos. The change in bar amplitude is 0.19 for the $q = 1.30$ model and 0.11 for the $q = 0.70$ model. The difference between the bar formation time and the re-growth time (by re-growth time, we mean the time when bar starts recovering from the weakening caused due to buckling) is longer for prolate halos as compared to oblate halos. This difference is 0.55 Gyr for the $q = 1.30$ model halo and 0.15 Gyr for the $q = 0.70$ model.

5.3.2   Bar buckling in oblate and prolate halos:

All of our model galaxies go though the buckling event during which the bar bends out of the disc plane and and becomes overall weaker. To verify that the buckling instability takes place, we have shown the edge-on view of the bars in the left column of Fig. 5.3 at the time of maximum buckling/bending amplitude (hereafter, we use t_{mb} to represent this time) calculated using equation 5.6. The right column of this figure shows the buckling induced boxy/peanut/x-shape pseudo-bulge after 5 Gyr of the maximum buckling amplitude (t_{mb}+5 Gyr). The direction of the bending depends on the mass asymmetry around the mid plane of the disc before buckling, and so the vertical direction can vary from model to model (Friedli & Pfenniger, 1990).

In Fig. 5.4, we have shown the color coded radial and temporal distribution of buckling amplitude (panels in top three rows) and boxy/peanut strength (panels in bottom row) in oblate, spherical and prolate halos. We have also over-plotted each panel with the temporal distribution of the maximum strength. Each column of this figure represents a model galaxy as shown at the top. The panels in the first row show the kinematic amplitude of bar buckling events while the second and third rows quantify morphological asymmetry caused in the bar due to buckling. Here all the high amplitude (i.e. distinct and measurable) peaks correspond to a buckling event during evolution. The fourth row at the bottom demonstrate the strength of boxy/peanut/x-shape structure induced by bar buckling.

As, in Fig. 5.2, we see a variation in the bar formation time with halo shape. The time of the bar buckling also varies with halo shape. From the first peaks of blue curves in the top row of Fig. 5.4, One can notice that the buckling timescale increases with decreasing halo shape parameter q. Prolate halos buckle earlier and oblate halos buckle later with respect to spherical halo. The kinematic signature of buckling indicates that oblate halos start buckling just after bar formation, whereas their prolate counterparts take some time to buckle after bar formation. On the other hand, the first buckling event attains its maximum amplitude just before bar re-growth time. These two effects result in closely similar buckling period during first buckling as can be interpreted from the width of the first peak in all the panels of the first row.

By comparing the panels of the first row in Fig. 5.4, one can clearly notice that the kinematic signature of first buckling is always higher in amplitude than the succeeding buckling events. However, the morphological signatures have higher amplitude during the second buckling event

Fig. 5.3 Bar buckling in non-spherical dark matter halos along with iso-density contours. Left column shows the side-on view (perpendicular to bar) of the disc at the time of maximum buckling amplitude (t_{mb}) and right column shows the side-on view after 5 Gyr of maximum buckling.

Fig. 5.4 Quantification of bar buckling and resultant BPX structure in non-spherical dark matter halos. First row panels show the buckling amplitude using stellar kinematics while second and third rows panels represent buckling amplitude using stellar morphology. Last row panels show BPX strength. Each column shows a different galaxy model. The temporal variation of maximum strength is shown by over-plotted blue curve.

as can be seen in the second and third row panels of the figure. When moving from oblate to prolate halos in any of the first three rows, we can spot a clear difference of increasing buckling signature at outer radial positions. This implies that prolate halos help the bar to buckle in the outer parts of the bar while oblate halos suppress the buckling at outer edges. At the outer edge of the bar, the buckling strength of the q1.15 model is slightly weaker than the q1.00 model. But it is strongest for the q1.30 model which indicates that the small deviation from spherical shape towards prolateness decreases the buckling amplitude but more deviation increases it. Prolate halo for q=1.30 shows an explicitly distinct buckling strength at the outer edges of the bar in both kinematic and morphological quantification which qualifies it to be a third buckling event. The spherical halo with q=1.00 also shows a weak signature of the third buckling event. In Fig. 5.5, we have shown the third buckling event in our most prolate halo and compared it with the spherical model. During the third buckling event, the iso-density contours in the prolate halo show clear noticeable distortion in the disc while the corresponding distortion in the spherical halo is very weak. Hence, the most prolate halo in our set of models has a halo shape that promotes the onset of three buckling events in the bar.

Seeing the widths of the peaks in each panel of the top three rows of Fig. 5.4, one can easily interpret that the duration of a preceding buckling is always smaller than the succeeding one. Since, the signatures of first buckling are short lived so, it has a lower probability to be

Fig. 5.5 Triple bar buckling events in q=1.30 halo along with iso-density contour compared with the q=1.00 halo. Each panel is shown at the time of distinct peaks in A_{buck,v_z} as shown by blue curves in the first row of Fig. 5.4. A clear bending of the iso-density contour can be noticed in the outer bar of the prolate halo model during 3rd buckling event.

detected in observations of galaxies. Instead it is the second buckling event which has a higher probability of being detected as an ongoing buckling event. In our models, the prolate halo of q=1.30 show a remarkable third episode of buckling, as can be seen in the last column of the figure. This suggests that in observations of galaxies, buckling events are more likely to be associated with prolate halos rather than oblate or spherical halos.

5.3.3   Effect of halo shape on the boxy/peanut bulge:

The last row of Fig. 5.4 shows the evolution of the boxy/peanut pseudo-bulge in distorted halos. There is one to one correlation between the bar buckling event and the steep (or sudden) rise in BPX strength. After each buckling event, the bar quickly gets thicker as can be seen in each panel of the bottom row. There is no significant difference in the inner bar BPX strength for different halos. But, the outer part of the bar in prolate halos gets more thicker than the oblate halos. The same result can be interpreted from the closer look at iso-density contours in the right column of Fig. 5.3. It shows that oblate halos try to restrict bar/disc against vertical thickening, whereas prolate halos promote vertical heating by the mean of continuous buckling.

The blue curves in the bottom row panels of Fig. 5.4 represent the time evolution of the maximum BPX strength in the shown region of the disc. Thus, for the sake of better

Fig. 5.6 Evolution of boxy/peanut/x-shape (BPX) structure in three different regions of the disc. For the sake of better visualization, only three extreme models (q0.70, q1.00, q1.30) are shown with different colors. Here, solid curves show central region $R \in [0, R_s)$, dashed curves represent middle region $R \in [R_s, 2R_s)$, and dotted curves demonstrate outer region $R \in [2R_s, 3R_s)$ as marked with symbols 'c', 'm', and 'o' respectively in the legend.

visualization, we have also shown the strength of BPX pseudobulges in three different parts of the disc for the three extreme models q0.70, q1.00 and q1.30 in Fig. 5.6. In the figure the solid curves show the central region $R \in [0, 1R_s)$, dashed curves represent middle region $R \in [1R_s, 2R_s)$, and dotted curves demonstrate outer region $R \in [2R_s, 3R_s)$. These regions are marked with symbols 'c', 'm', and 'o' respectively in the legend of the figure. The solid curves of Fig. 5.6 clearly show that the central part of the bar that lies within disk scale radius thickens least and the amplitude is fixed in all the models. Similarly, the middle part of the bar shows nearly equal BPX amplitude in all the models after evolution but its magnitude is higher than that of the central part of the bar. The outer bar region shows more thickening than the central and middle regions for all the models. But the outer region also shows the maximum thickening for the prolate halo as compared to the spherical and oblate halos.

5.4   Conclusions and Discussion

We have performed N-body simulations of bar buckling in isolated disc galaxies with different dark matter halo shapes ranging from oblate to prolate. The Fourier analysis techniques are

used for the quantification of the bar and buckling strength. The main findings of this work are listed below.

(i) Oblate halos delay the bar formation and provide support to the bar against the weakening due to buckling instability, whereas prolate halos helps in early bar formation and this bar weakens more during buckling. But, the re-growth of the bar does not leave any signature of halo shape in bar amplitude. Bar strength become same in both types of halos after recovering from first buckling.

(ii) Though the bar in the prolate halo takes a longer time to re-grow than the bar in an oblate halo, the time period of first buckling event remains closely similar. This is because prolate halos take extra time to buckle after bar formation but oblate halos buckle just after bar formation. On the other hand, introducing small prolateness in the spherical halo suppresses the third buckling event that is seen in the spherical halo but, increasing more prolateness promotes the third buckling event while oblate halos always suppress third bar buckling. In our models, all the oblate halos show only two distinct buckling events. However, the most prolate halo in our set of simulations, q=1.30, displays three noticeable well defined buckling events.

(iii) The buckling induced boxy/peanut/x-shape structure (or BPX bulge) remains closely similar in the inner part of the bar irrespective of the dark matter halo shape. But, outer edge of the bar thickens more in prolate dark matter halos as compared to oblate and spherical dark matter halos. Prolate halos make bar/disc more thicker than the oblate and spherical dark matter halos.

Oblate halos provide more gravitational potential near the disc plane as compared to prolate halos (see Fig. 5.1). As a result, the rotation velocity, Toomre instability parameter, and vertical dispersion of the disc particles increases and bar formation is delayed in oblate halos. In our set of simulations, the most prolate halo with shape parameter q=1.30 shows the noticeable third buckling event and the spherical model shows only a very weak third buckling event. All the oblate halos and the halo with small prolateness suppress the signature of third buckling event. On the other hand, the timescale of first and second buckling events are very small as compared to the third one. This itself explains why the detection of ongoing buckling events in the observations of nearby galaxies is such a small fraction of all barred galaxies. Till date only 8 buckling events have been observed (see Table 2 in Xiang et al. (2021)) using various existing methodologies.

Our simulations do not account for the presence of gas, classical bulge and interactions with other galaxies. All these factors also affect the bar formation and buckling of the bar. The presence of gas, and classical bulge prevent bar formation whereas flyby interactions have been seen to produce bars (Łokas, 2018; Smirnov & Sotnikova, 2019; Łokas, 2020). Studying the effect of various components of a galaxy on bar buckling and the galaxy environment,

may provide constraints on the shapes of dark matter halos. O'Brien et al. (2010) and many previous studies suggest that dark matter halos shape parameter lies in the range of $q=0.1$ to $q=1.4$. A detailed statistical study of halo shapes in observations of galaxies and state of art cosmological simulations may help improve our understanding of bar buckling and the formation of boxy/peanut/x-shape structures.

"We all travel the Milky Way together, trees and men."

– **John Muir**

6

Growth of Disc-like Pseudo-bulges in SDSS DR7 Since $z = 0.1$

6.1 Introduction

The galaxy evolution is broadly governed by two kind of processes namely a) gravitational clustering i.e. collapses, merger events and b) internal secular processes like bar, spiral arms etc (Norman et al., 1996; Conselice, 2014). It is well known that internal secular evolution of disc galaxies leads to significant changes in the properties of the bulges (Kormendy & Kennicutt, 2004b; Combes, 2009). Therefore, in order to understand the galaxy formation and evolution processes, the study of bulges is quite insightful. The nature of different type of bulges has been explored in simulations (Athanassoula, 2005b) as well as in observations (Fisher & Drory, 2008b, 2011b; Erwin et al., 2015). Morphological studies of galaxies show that the bulge to disc ratio varies from early to late type spiral galaxies in the Hubble sequence (Laurikainen et al., 2007; Graham & Worley, 2008).

There are broadly two type of bulges given the recent understanding; classical and disc-like bulges. These are differentiated with the help of photometric, kinematic properties and stellar population of the stars they possess (Athanassoula, 2005b; Fisher & Drory, 2008b;

Athanassoula, 2016b; Laurikainen & Salo, 2016b). Classical bulges are thought to be formed in major mergers (Kauffmann et al., 1993b; Baugh et al., 1996; Hopkins et al., 2009c; Naab et al., 2014), accretion of smaller satellites (Aguerri et al., 2001b), multiple minor mergers (Bournaud et al., 2007a; Hopkins et al., 2010a), and monolithic collapse of a primordial cloud (Eggen et al., 1962). Classical bulges are rounder objects like elliptical galaxies and contains older population star with higher velocity dispersion compare to disc stars (Kormendy & Kennicutt, 2004b). On the other hand, disc-like bulges are flattened systems like an exponential disc in the nuclear region Athanassoula (2005b). They are thought to be formed by the inward pulling of gas along the orbits and the consequent star formation (Kormendy, 1993b; Heller & Shlosman, 1994a; Regan & Teuben, 2004). These disc-like bulges are also known as pseudo-bulges (Kormendy & Kennicutt, 2004b) which were conceptualized by Kormendy (1982, 1983).

Now, the term pseudo-bulge is commonly used to describe disc-like bulges and boxy/peanut structures seen in edge-on galaxies (Athanassoula, 2005b). Boxy/peanut structure are vertically thick systems and are dominated by rotational motion of the stars similar to the disc-like bulges. However, it is well-established that Boxy/Peanut structures are just bars seen under different galaxy inclinations (Bureau & Freeman, 1999; Lütticke et al., 2000). They are thought to be formed by disc instability during secular evolution (Kormendy & Kennicutt, 2004b), vertical heating of the bar due to buckling (Combes et al., 1990b; Raha et al., 1991a; Martinez-Valpuesta et al., 2006a; Shen et al., 2010; Kataria & Das, 2018; Kumar et al., 2022a), or heating of the bar due to vertical resonances (Pfenniger & Norman, 1990). Classical bulges are very stable against galaxy flybys but boxy/peanut structures grow significantly in major flybys (Kumar et al., 2021). Both, disc-like pseudo bulges and box/peanut structures, are very different in terms of physical properties and formation mechanism (Athanassoula, 2005b; Laurikainen & Salo, 2016b). To be more specific, we focus our study on classical bulges and disc-like pseudo-bulges (hereafter, we refer disc-like pseudo-bulge as pseudo bulge in through out the draft.)

Magneto-hydrodynamics zoom-in cosmological simulations "Auriga simulations" with sub-grid physics (Gargiulo et al., 2019) show that pseudo-bulges are prominent in Milky Way type halos. In the same study, it has been shown that around 75 % of the bulges in these simulations have in-situ stars which are formed around $z = 0$ rather than in accretion events. Fisher (2006d) used the PAHs (polycyclic aromatic hydrocarbons) surface brightness profiles and found that the star formation mechanisms for classical bulges and pseudo-bulges are very different. Star formation in classical bulges is fast episodic, whereas pseudo-bulges show long lasting star formation. These studies resonate with semi-analytical modeling of L-galaxies in Millenium and Millenium II simulations (Izquierdo-Villalba et al., 2019) which has shown the quiet merger history for pseudo-bulged galaxies compare to classical one.

Laurikainen et al. (2007) has shown that pseudo-bulges are widespread along all the Hubble sequence galaxies like classical bulges. It has been also pointed out that pseudo-bulges are found mostly in galaxies with lower bulge to total mass ratio (B/T), and galaxies with $B/T \geq 0.5$ claimed to have classical bulges surely (Kormendy & Fisher, 2005). Several observational studies (Weinzirl et al., 2009b; Kormendy et al., 2010a) find that the fraction of pseudo-bulges in their nearby galaxies sample is larger than 0.5. This fact has raised questions regarding hierarchical structure formation scenario under standard ΛCDM cosmology (Kormendy et al., 2010a).

As we have seen the evolution of pseudo-bulges with cosmic time still remains a mystery. We are motivated to ask questions such as "How do the pseudo-bulge and classical bulge fraction varies with redshift?", "How do the pseudo-bulge properties varies with redshift?". In this article, we study the evolution of bulges where we want to look at the frequencies of the two types of bulges with redshifts. This will surely lead us to understand the connection between two types of bulges.

The plan of the chapter is as follows. In Section 6.2, we have mentioned about the complete sample and our selection criterion along with the $N-$body modeling of galaxies. The evolution of the bulges, their shapes, correlation with various photometric parameters of galaxies, and comparison with local volume survey are shown in Section 7.5. The effect of telescope resolution and disc inclination using simulated galaxies is explored in Section 6.4. The effect of data selection criteria on our results is described in Section 6.5. In Section 6.6, implication of this study are discussed. The brief concluding summary is pointed out in Section 8.1.

6.2 Data and Sampling

6.2.1 Complete Data

In this work, we use the archival data from Simard et al. (2011). The detailed information of data and analysis can be found in Simard et al. (2002, 2011) but, for the benefit of the reader, we are describing in brief about the data. Simard et al. (2011) provides the two-dimensional decomposition of 1.12 million objects in g- and r-band from Legacy area of Sloan Digital Sky Survey Data Release Seven (SDSS DR7) (Abazajian et al, 2009). Morphologically, these objects are galaxies and have galactic extinction corrected r-band Petrosian magnitude in the range 14 to 18. The structural parameters of the galaxies were determined using 2D decomposition tool GIM2D (Simard et al., 2002). Only two components, Sérsic bulge and exponential disc, were used for the decomposition of all the galaxies in the sample. The data is available in three formats of the fittings: (1) fixed Sérsic index $n = 4$ + disc fitting, (2) free

Fig. 6.1 Complete data of galaxies and selected data for this study from Simard et al. (2011). Data distribution from left to right, top row: spectroscopic redshift (z), r-band absolute magnitude of galaxies ($M_{r,g}$), physical disc scale radius (R_d); bottom row: stellar mass of galaxies ($M_{*,g}$), r-band apparent magnitude of galaxies ($m_{r,g}$), and apparent disc scale radius (R_d). Black and red histograms show complete data and selected data respectively. Large difference in complete data and selected data is the result of our stringent selection criteria which mostly remove faint and low-mass galaxies.

Sérsic index from 0.5 to 8 + disc fitting, and (3) free Sérsic index from 0.5 to 8 fitting. For the calculation of physical parameters of the galaxies, the cosmological parameters $H_0 = 70$ km s^{-1} Mpc^{-1}, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ are used.

We also used the archival data from Salo et al. (2015). The detailed information of data and methodology is described in Muñoz-Mateos et al. (2015); Salo et al. (2015). They provide the two-dimensional decomposition of 2352 galaxies at 3.6 μm (mid-infrared) form Spitzer Survey of Stellar Structure in Galaxies (S^4G) (Sheth et al., 2010). The structural parameters of the galaxies were calculated using 2D decomposition tool GALFIT (Peng et al., 2002a, 2010a). Five components, exponential disc, edge-on disc, Sérsic bulge, ferrer bar, and unresolved central component psf (point spread function), were used to decompose the galaxies whenever required. The data is available in two formats of the fitting: (1) multi-component fitting, and (2) free Sérsic index from 0.3 to 19 fitting.

6.2.2 Our Criteria

To understand the evolution of bulges in disc galaxies, we will use the free Sérsic index from 0.5 to 8 + disc fitting of Simard et al. (2011) (hereafter SDSS data) and multi-component fitting from Salo et al. (2015) (hereafter S^4G data). The redshift or distance of nearby galaxies is severely contaminated by their peculiar velocities. Therefore, we have excluded a small fraction of galaxies with $z < 0.005$ as suggested in Shen et al. (2003). We have limited our study to maximum redshift of $z = 0.1$, and spectroscopically confirmed galaxies which are best fitted by two components. We have avoided all the galaxies with apparent bulge effective radius and disc scale radius equal or smaller than the psf radius as these galaxies will have more uncertainties in the photometric parameters. Next we rejected fainter ($m_r > 17$) and smaller galaxies ($M_* < 10^9$ $M_\odot$) from our sample. Stellar mass of the galaxy (M_*) was estimated using color-stellar mass-to-light ratio relation (CMLR). Stellar mass-to-light ratio (γ_*) of galaxies is related to the color of galaxies by the following expression,

$$\log \gamma_*^j = a_j + b_j \times color, \tag{6.1}$$

where a_j and b_j are two constants calculated for j^{th} imaging band at given color index (e.g. $g - r$ in our case). To calculate r-band stellar mass-to-light ratio, We have adopted values of a_j and b_j from Bell et al. (2003). Recently, Du & McGaugh (2020) have re-calibrated these constants for the galaxy mass consistency in various stellar mass-to-light ratio estimators. Both studies give same constants in r-band for $g - r$ color. After all these constraints, we end-up with a total sample of 105,160 galaxies. Hereafter, we refer this sample to parent data.

To minimize the uncertainties in our results, we have imposed the strict constraint of maximum statistical error of 10% on the magnitude of each component, bulge Sérsic index, disc inclination, bulge to total light ratio, bulge ellipticity, and size of each component. We removed all the galaxies whose Sérsic index is 0.5 or 8.0 and bulge ellipticity is 0.0 or 0.7 to avoid the fitting bias in the sample due to presence of bar, close merger, point source etc (Simard et al., 2011). Finally, we constrain our sample to the galaxies where half-light radius of the bulge is smaller than the half-light radius of the disc. These constraints reduce our parent data of 105,160 galaxies to 40,504 galaxies.

Since we are interested in the evolution of the bulges, we need to take care of barred galaxies in the sample. To remove the barred galaxies from our sample, we used the catalog of "Galaxy Zoo 2" classification (Willett et al., 2013; Hart et al., 2016). Galaxy Zoo is a citizen science project, where images of the galaxies are shown to the interested citizens and they are asked to answer some basic questions related to the galaxy morphology. These answers are used for morphological classification of the galaxies depending on the probability of the voting. Though

this method does not removes all the barred galaxies, but it improves our sample. Removal of barred galaxies reduces our sample from 40,504 to 38,996 galaxies. Hereafter, we refer this sample to selected data.

For a comparison between our selected data and the parent data of galaxies, we have shown the number distribution of galaxies in Fig. 6.1. From left to right, top row panels: spectroscopic redshift (z), r-band absolute magnitude of galaxies ($M_{r,g}$), physical disc scale radius (R_d); bottom row panels: stellar mass of galaxies ($M_{*,g}$), r-band apparent magnitude of galaxies ($m_{r,g}$), and apparent disc scale radius (R_d). Physical size of galaxy was calculated using its angular distance as mentioned in Simard et al. (2011). Our selected data represents the fair sample of the parent data except at faint and low-mass end of the distribution. It is because our strict constraint of maximum 10% statistical error in each photometric parameter. Later, in Section 6.5, we will discuss the effect of this strict selection criteria on our results.

In our analysis, we have divided data in several different categories. The disc dominating and bulge dominating galaxies are defined on the basis of bulge to total light ratio B/T in r-band. The disc dominating galaxies are those which have $B/T \leq 0.5$ and the bulge dominating galaxies have $B/T > 0.5$. None of the photometric classification of the bulges provides proper separation between classical bulges and pseudo-bulges. Particularly, $n = 2$ Sérsic index does not show bi-modality in the distribution of the bulges (Graham, 2013, 2014; Costantin et al., 2018; Méndez-Abreu et al., 2018; Gao et al., 2020; Kumar et al., 2021). Some elliptical galaxies, and bulges of S0 and merger built galaxies also show lower Sérsic indices (Davies et al., 1988; Young & Currie, 1998; Eliche-Moral et al., 2011; Querejeta et al., 2015; Tabor et al., 2017). In Section 6.4, We have also verified using numerical models of Milky Way mass galaxies that Sérsic index is not reliable for bulge classification, given the inclination of the disc and resolution of the telescope. Hence, for a more precise classification of the bulges, we have adopted a combination of Sérsic index based classification and Kormendy relation based classification, which we call Sérsic-Kormendy classification. Sérsic index based classification (hereafter Sérsic classification) makes use of $n = 2$ Sérsic index. All the bulges having $n \leq 2$ are categorized as pseudo-bulges, whereas bulges with $n > 2$ are categorized as classical bulges (Fisher, 2006b; Fisher & Drory, 2008b). On the other hand, Kormendy relation based classification (hereafter Kormendy classification) uses well known Kormendy relation for elliptical galaxies to separate two types of bulges. All the bulges which lie above 3σ limit of the Kormendy relation are grouped as classical bulges, while the bulges which fall below 3σ limit are grouped as pseudo-bulges (Gadotti, 2009b). For this study, we have taken Kormendy relation from Lackner & Gunn (2012). In Sérsic-Kormendy classification, only those bulges are considered which satisfy both Sérsic and Kormendy classifications simultaneously. For

the sake of comparison, we will also be showing our results using Sérsic classification and Kormendy classification along with Sérsic-Kormendy classification.

6.2.3 Simulated Galaxy Models

To understand the effect of spatial resolution of telescope, projection of galaxy, and surface density of galaxy on fitting parameters, particularly Sérsic index, we simulate model galaxies with bulge to disc mass ratio (B/T) 0.1, 0.3, 0.5, and 0.7 keeping fixed disc mass and fixed bulge scale radius. There are four disc surface densities for each model. The total mass (stellar bulge + stellar disc + dark halo) of each the model galaxy is similar to that of Milky Way type galaxy. We use the publicly available $N-$body code GALIC (Yurin & Springel, 2014) to generate our model galaxies. In our models, dark matter halo density is represented by Hernquist profile,

$$\rho_{dm}(r) = \frac{M_{dm}}{2\pi} \frac{a}{r(r+a)^3} \tag{6.2}$$

where 'a' is dark matter halo scale radius. This is related to the concentration parameter 'c' of a corresponding NFW halo (Navarro et al., 1996) of mass $M_{dm} = M_{200}$ by the following expression,

$$a = \frac{r_{200}}{c} \sqrt{2\left[\ln(1+c) - \frac{c}{(1+c)}\right]} \tag{6.3}$$

where r_{200} is the virial radius of galaxy. This is the radius within which the average matter density is 200 times the critical density of the Universe. M_{200} is mass within the virial radius.

The disc density decays exponentially in the radial direction and its vertical distribution is described using sech^2 profile

$$\rho_d(R,z) = \frac{M_d}{4\pi z_0 R_s^2} \exp\left(-\frac{R}{R_s}\right) \text{sech}^2\left(\frac{z}{z_0}\right) \tag{6.4}$$

where M_d is total disc mass, R_s is disc scale radius, and z_0 is disc scale height.

The bulge density is also modelled by the spherically symmetric Hernquist density profile

$$\rho_b(r) = \frac{M_b}{2\pi} \frac{b}{r(r+b)^3} \tag{6.5}$$

where M_b represents the total bulge mass and 'b' represents the bulge scale radius.

We set 1 million particles for each component which results in a total of 3 million particles in each model galaxy. The rotation velocity of each model galaxy is set to be 220 km s^{-1} which corresponds to a total mass $2.47 \times 10^{12} M_\odot$ (where $M_\odot$ = mass of the Sun). The disc mass is

fixed at 0.04 of the total galaxy mass. The gravitational softening is set to be 0.01 kpc for each type of particle.

For the purpose of bulge-disc decomposition of the simulated galaxies, we use the latest version of GALFIT (Peng et al., 2002a, 2010a) which is a widely used for 2d decomposition of galaxies. To see the effect of the spatial resolution, the pixel size is set to be 0.025 $kpc \times$ 0.025 kpc (hereafter high-resolution) and 0.05 $kpc \times$ 0.05 kpc (hereafter low-resolution). For the decomposition of simulated galaxies, we did not consider any background sky and the point spread function (psf) is taken to be delta function.

6.3   Results

6.3.1   Distribution of Bulges with Redshift

The evolution of the classical and pseudo-bulges over the cosmic time from redshift 0.005 to 0.1 is represented in the Fig. 6.2 from SDSS survey. The effect of classification method on the distribution of the bulge type is shown in the left, middle, and right columns of the figure which are corresponding to the Sérsic classification, Kormendy classification, and Sérsic-Kormendy classification respectively. The contribution of galaxy type on the distribution of the bulge type is shown in the top, middle, and bottom rows of the figure which are the representative of all, disc dominating, and bulge dominating galaxies respectively. Red curves show fraction of classical bulges and green curves show fraction of pseudo-bulges. Statistical uncertainty in the fraction is shown using error bars and is estimated from $\sqrt{[f \times (1 - f)]/N}$, where f is the fraction of any point and N is number of objects at that point in the distribution (Sheth et al., 2008). The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. At many points, error bar is smaller than the size of the symbol. All the panels (except bottom left) of this figure indicate that the local Universe is dominated by pseudo-bulges. The high fraction of the pseudo-bulges is contrary to the prediction of cosmological simulations. According to widely accepted ΛCDM cosmological model, we expect more classical bulges instead of pseudo-bulges (White & Rees, 1978b; Aguerri et al., 2001b; Bournaud et al., 2005a; Baugh, 2006; Brooks & Christensen, 2016).

As we go back in the time towards the high-redshift Universe, the fraction of pseudo-bulge decreases and classical bulges start dominating over the pseudo-bulges. In the evolution of the Universe, there comes a time before that Universe was dominated by the classical bulges or spheroids. Since no bulge classification method shows perfect bimodality in two types of bulges, the precise redshift of equality in classical bulges and pseudo-bulges cannot be well constrained. Sérsic classification shows the point of equality a lower redshift than the

Fig. 6.2 Fractional distribution of the classical and pseudo-bulges with redshift (z) in the SDSS survey. Left, middle, and right columns show Sérsic, Kormendy, Sérsic-Kormendy classifications respectively. Top, middle, and bottom rows represent all, disc dominating, and bulge dominating galaxies respectively. Red and green curves represent classical bulges (CB) and pseudo-bulges (PB) respectively. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. All the methods show domination of pseudo-bulges over classical bulges with decreasing redshift.

Kormendy classification. As a consequence, Sérsic-Kormendy classification show equality in between the two classification. If we see the distribution of all the sample galaxies in Sérsic, Kormendy, and Sérsic-Kormendy classifications, the fraction of the classical and pseudo-bulge becomes equal at $z \approx 0.009, 0.020, 0.016$ redshifts respectively.

One can see the effect of bulge classification criterion on the distribution of bulge type in all, disc dominating, and bulge dominating galaxies by comparing the panels of top, middle, and bottom row respectively. All three rows show that the fractional distribution of the bulges in Sérsic classification is very different than the other two classifications. In the low-redshift region, it always shows small fraction of pseudo-bulges as compare to the Kormendy and Sérsic-Kormendy classifications. But, the trends of distribution are more or less similar in Kormendy and Sérsic-Kormendy classifications except for bulge dominating galaxies at low-redshift where Kormedy classification show more pseudo-bulges than the Sérsic-Kormendy classification.

Similarly, the contribution of galaxy type on the distribution of bulge type in Sérsic, Kormendy, Sérsic-Kormendy classifications can be seen by comparing the panels of left, middle, and right column respectively. From all three columns, it is clear that the disc dominating galaxies provide more pseudo-bulges than the bulge dominating galaxies in the fractional distribution of the bulges at low-redshift. This implies that most of the pseudo-bulges are low mass relative to their hosting discs. Hence, we conclude that the Kormendy and Sérsic-Kormendy classifications show quite similar distribution of the bulges but, Sérsic classification results in a lower fraction of pseudo-bulges at low-redshift. Kormendy classification show slightly higher fraction of pseudo-bulges than Sérsic-Kormendy classification at low-redshift in bulge domination galaxies. However, local volume remains pseudo-bulge domination irrespective to the classification scheme, and the dominating contribution of pseudo-bulges comes, mostly, from the disc dominating galaxies.

6.3.2   Distribution of Bulges with R_e/R_{hlr}

To see the effect of the bulge and disc sizes on the distribution of bulge type, we have calculated the fractional distribution of the classical and pseudo-bulges with the ratio of bulge semi-major half-light radius R_e to disc semi-major half-light radius (R_{hlr}) from SDSS survey in Fig. 6.3. The ratio of bulge semi-major half-light radius to disc semi-major half-light radius (R_e/R_{hlr}) is better than the absolute bulge or disc size because it removes the error, if any, in the size calculation using distance or redshift of the galaxy. However, in appendix 6.8, we have also shown the absolute sizes of the bulges and discs with redshift and compared them together. In this figure, columns show the effect of the classification method and rows show the effect of galaxy type on the distribution of bulge type. Left, middle, and right columns represent Sérsic,

Fig. 6.3 Fractional distribution of classical and pseudo-bulges with the ratio of bulge semi-major half-light radius (R_e) to disc semi-major half-light radius (R_{hlr}) in the SDSS survey. Left, middle, and right columns show Sérsic, Kormendy and Sérsic-Kormendy classifications respectively. Top, middle, and bottom rows represent all, disc dominating, and bulge dominating galaxies respectively. Red and green curves represent classical bulges and pseudo-bulges respectively. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. The fraction of pseudo-bulges increase with increasing bulge to disc size ratio.

Kormendy, and Sérsic-Kormendy classifications respectively. Top, middle, and bottom rows are the representatives of all, disc dominating, and bulge dominating galaxies respectively. From all the panels of the figure, it is clear that a large fraction of the small bulges (small R_e/R_{hlr}) is classical in nature. The fraction of pseudo-bulges increases with increasing R_e/R_{hlr} or vice-versa the fraction of classical bulges increases with decreasing R_e/R_{hlr}. Unitl $R_e/R_{hlr} \approx 0.6$, these trends are valid in all the panels.

The effect of bulge classification method on the distribution of bulge type in all, disc dominating, and bulge dominating galaxies can be seen by comparing the panels of the top, middle, and bottom rows respectively. All the three rows of the figure show that the Kormendy classification results in more pseudo-bulges than the other two classification methods for $R_e/R_{hlr} \geq 0.6$. Particularly, this difference is more distinct in disc dominating galaxies. However, the classification of bulges in bulge dominating galaxies does not shows significant change among three methods. In the same way, one can see the effect of galaxy type on the distribution of bulge type in Sérsic, Kormendy, and Sérsic-Kormendy classifications by comparing the panels of left, middle, and right columns respectively. All the columns of the figure clearly indicate that the contribution of disc dominating galaxies in the distribution of pseudo-bulges is always great or equal to that of the bulge dominating galaxies until $R_e/R_{hlr} \approx 0.6$. Hence, for $R_e/R_{hlr} \leq 0.6$, a large fraction of pseudo-bulges comes from the disc dominating galaxies. Inversely, a large fraction of classical bulges comes from the bulge dominating galaxies. These conclusions hold for all the classification methods discussed here. When moving towards larger value of R_e/R_{hlr}, disc dominating galaxies start showing increasing classical bulges in Sérsic and Sérsic-Kormendy classifications. But, the Kormendy classification still shows increasing pseudo-bulges in disc dominating galaxies. According to Hubble classification, S0 galaxies have massive and large bulges which are comparable to their discs. These bulges are generally classical in nature. Therefore, we expect increase in the fraction of classical bulges close to $R_e/R_{hlr} = 1$ which is not coming out in Kormendy classification. However, Sérisc-Kormendy classification captures it well.

For a given bulge to total light ratio (B/T), the distribution of the bulges with R_e/R_{hlr} follows more or less similar trends to that of Fig. 6.3. This implies that the ratio R_e/R_{hlr} can be treated as an indicator of the ratio of disc to bulge mean surface densities (see the appendix 6.9). For example, we can say that the bulges with $R_e/R_{hlr} < 0.5$ are more concentrated (or dense) than the bulges with $R_e/R_{hlr} > 0.5$ relative to their hosting discs. From all the panels of the Fig. 6.3, we can see that the preferential condition for the bulges to be pseudo is their low concentration relative to the hosting discs. Concentrated bulges are usually classical in nature. Hence the relative surface densities of the bulge and disc play a crucial role in the formation and evolution of the bulges. A low surface density disc usually lacks from global instabilities

in its center (Toomre, 1964; Mihos et al., 1997; Mayer & Wadsley, 2004; Cervantes Sodi & Sánchez García, 2017; Peters & Kuzio de Naray, 2019). So, most probably, it will also lack from the pseudo-bulge.

6.3.3   Distribution of Bulge Ellipticity with Redshift

Classical bugles are supported by the random motion of stars that makes them more round in shape as compare to the pseudo-bulges which are supported by the ordered rotational motion of the stars. Rotational motion of the stars in pseudo-bulges provide them flat shape. In its evolution, galaxy goes through several dynamical and morphological changes due to secular evolution and interactions with other galaxies (Kumar et al., 2021). To trace the morphological changes in the bulges during the evolution of the galaxies, we calculated mean ellipticities ($< e >$) of the classical bulges and pseudo-bulges as a function of redshift. For this purpose, we deprojected all the bulges to minimize the effect of disc inclination on projected shape of the bulges (mainly pseudo-bulges). Fig. 6.4 shows the evolution of the mean bulge ellipticity of classical bulges and pseudo-bulges with redshift for SDSS data. Red and green curves represent classical bulges and pseudo-bulges respectively. The standard dispersion from mean is displayed using shaded regions around mean values, and statistical uncertainty due the number of galaxies is shown with vertical bars. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. We have demonstrated Sérsic, Kormendy, and Sérsic-Kormendy bulge classification schemes in first, middle, and right columns respectively. Rows, starting from the top, show the ellipticity evolution in all, disc dominating, bulge dominating galaxies respectively.

One thing we can clearly notice from this figure is that the mean ellipticity of the pseudo-bulges is always higher than the classical bulges irrespective to the classification scheme and redshit. It means that the classical bulges are generally rounder than the pseudo-bulges when seen in face-on projection of galaxies. On the other hand, all the panels show decreasing mean ellipticity with decreasing redshift for both types of bulges (when considering points with statistically significant count of galaxies for reasonable mean ellipticity). The reducing mean ellipticity of the bulges implies that the both types of bulges are getting rounder and rounder with the evolution of the galaxies.

The effect of bulge classification scheme on the evolution of mean bulge ellipticity in all, disc dominating, and bulge dominating galaxies can be marked by comparing all the panels of top, middle, and bottom rows respectively. From all the rows, we can see that Kormendy and Sérsic-Kormendy classification show quite similar declining trends in mean ellipticity for both types of bulges. Sérsic classification method exhibits shallower decline in mean ellipticity of classical bulges than the other two classification methods. But, at low-redshift, the ellipticity of

Fig. 6.4 Distribution of mean deprojected bulge ellipticity ($< e = 1 - b/a >$) with redshift in the SDSS survey. Left, middle, and right columns show Sérsic, Kormendy, and Sérsic-Kormendy classifications of bulges respectively. Top, middle, and bottom rows represent all, disc dominating, and bulge dominating galaxies respectively. Red, and green color curves represent the distribution of classical bulges, and pseudo-bulges bulges respectively. Shaded regions show standard deviation from mean and vertical bars represent statistical uncertainty due to number of galaxies. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. All panels show that bulges are moving close to the round shape with decreasing redshift.

pseudo-bulges remains nearly unaffected in three classification methods. In the similar manner, one can observe the effect of galaxy type on the distribution of bulge ellipticity in Sérsic, Kormendy, and Sérsic-Kormendy classifications by comparing all the panels of left, middle, and right columns respectively. From all the columns, one can clearly see that both types of bulges in bulge domination galaxies show lower mean ellipticity than the disc dominating galaxies. It is true for whole redshift range and for three classifications when considering points with statistically significant number of galaxies. Also, the rate of declining mean ellipticity is steeper in bulge dominating galaxies than in disc dominating galaxies. Now, we can conclude that the shape of massive bulges is more axisymmetric than the low-mass bulges at whole redshift range. Also, the high-mass pseudo-bulges are moving rapidly towards the axisymmetry.

One should note that we have removed the sample of barred galaxies in our analysis. The observed fraction of the barred galaxies ranges from 30% to 60% in the local volume (Aguerri et al., 2009; Masters et al., 2011; Cervantes Sodi et al., 2015; Díaz-García et al., 2016). This fraction declines when we move from low-redshift to high-redshift (Sheth et al., 2008; Melvin et al., 2014). Therefore, it may possible that some galaxies with bulges had formed bar during their evolution. The conversion of bulged galaxies into barred galaxies can affect the mean ellipticity of the bulges. But, how much it will influence evolution of the bulges remains the question for further study.

6.3.4 Evolution of the Bulges with Galaxies

The different components of the galaxies evolve together with the evolution of the Universe. Therefore, it will be interesting to understand the evolution of the photometric properties of the galaxies with the redshift and their correlations with the evolution of the bulges. In the Fig. 6.5, we have shown the distribution and evolution of bulge to total light ratio (B/T), disc inclination (i), bulge ellipticity (e), r band galaxy absolute magnitude ($M_{r,g}$), bulge Sérsic index (n), and bulge to disc half light radius ratio (R_e/R_{hlr}) with redshift (z) from SDSS survey. The color of the distribution represents the number density of galaxies on log scale. The red solid curves represents the median of the data and shaded color show the $\pm 25\%$ dispersion of the data from median.

From the first column of the first row, one can see that the median of bulge to total light ratio (B/T) is decreasing with decreasing redshift. The Universe is becoming disc dominating (dynamically cool) as it is getting older and older. In the local Universe, median + 25% curve is below the $B/T = 0.5$ cut-off i.e. matter fraction in disc component is higher than the bulge component. This suggests that nearly 75% of the local Universe is dynamically cool. Similar signatures of dynamical cooling can be found from the second and third columns of the first row which show that the medians of inclination (i) is nearly constant however the bulge ellipticity

Fig. 6.5 Distribution and evolution of different photometric properties of the galaxies with the redshift (z) from SDSS survey. On the y axis, B/T = bulge to total light ratio, i = disc inclination, e = bulge ellipticity, $M_{r,g}$ = r band galaxy absolute magnitude, n = bulge Sérsic index, and R_{e}/R_{hlr} = bulge to disc half-light radius ratio. The number density of the galaxies is represented by the color on log scale. The red solid curves show the median of the data and shaded region represents the $\pm 25\%$ dispersion of the data from median.

(e) is increasing with decreasing redshift. Increase in the bulge ellipticity at the constant disc inclination is the indication of the deviation of the bulges from their classical nature. Note that the less number of data points near zero ellipticity in third column of first row is the result of our tight criteria of maximum 10% error in any quantity. Also, the increase in median ellipticity here should not be confused with decrease in mean ellipticity in Fig. 6.4. Here, we have considered inclined galaxies.

First column of the second row shows that the median of r band galaxy absolute magnitude ($M_{r,g}$) is increasing with decreasing redshift. These two quantity, magnitude and redshift, have strong and obvious correlation because at a given redshift/distance, we cannot observe an object fainter than the limit of the telescope. Therefore, we are seeing increased number of the fainter galaxies with decreasing redshift. The second column of the second row illustrates the decreasing median of Sérsic index (n) with decreasing redshift. This also indicates the dynamical cooling of the galaxies with the evolution of the Universe. The third column of the second row shows the evolution of the bulge to disc semi-major half light radius ratio (R_{e}/R_{hlr}) with redshift. Though the change is very small, but it is increasing linearly indicating either faster growth of the bulges than the discs or the growing ellipticity of the bulges.

We have also calculated the Pearson's linear correlation coefficient (cc) of the bulge Sérsic index (n) with other photometric properties of the galaxies in different redshift bins of the

Fig. 6.6 Linear correlation coefficient of the bulge Sérsic index (*n*) with different photometric properties of the galaxies in different redshift bins of SDSS survey. The blue, orange, green, and red color solid curves represent the linear correlation of bulge Sérsic index with bulge to total light ratio (B/T), disc inclination (*i*), bulge ellipticity (*e*), and r band galaxy absolute magnitude ($M_{r,g}$) respectively. The number of galaxies at any point are also shown at respective redshift.

SDSS survey. The linear correlation coefficient of two variables tells how the one variable changes with the change in the other variable. Its value lies between -1 (for a perfect negative correlation) and +1 (for a perfect positive correlation). In the Fig. 6.6, the blue, orange, green, and red color solid curves represent the linear correlation of bulge Sérsic index with bulge to total light ratio (B/T), disc inclination (*i*), bulge ellipticity (*e*), and r band galaxy absolute magnitude ($M_{r,g}$) respectively. At high-redshift, B/T has very weak positive correlation. It weakly develops positive correlation with decreasing redshift. Disc inclination has very weak and nearly constant anti-correlation at all redshifts. Bulge ellipticity and absolute galaxy magnitude show the weakly increasing negative correlation as we move towards the low-redshift Universe. However, they show sudden decrease in the negative correlation at very low-redshift. Only bulge ellipticity and absolute galaxy magnitude show significant, but small, anti-correlation just before the sudden drop at low-redshift.

6.3.5   Comparison with Local Volume Survey

The SDSS data provides two-component fittings of the galaxies. But, in reality, a galaxy can have more or less than two components. Therefore, two-component fittings can provide biased parameters of the galaxies which do not have exactly two components (Aguerri et al., 2005;

Fig. 6.7 Comparison of SDSS survey with S^4G survey. Top panel shows the Sérsic index histogram and bottom panel shows the bulge ellipticity histogram of 27 galaxies, common in both survey, which are fitted by Sérsic component. The red and blue filled histograms show the galaxies from SDSS and S^4G survey respectively and vertical lines represent the medians of respective data. SDSS data over-estimates Sérsic index and under-estimates bulge ellipticity.

Laurikainen et al., 2005; Gadotti, 2009b; Weinzirl et al., 2009b; Méndez-Abreu et al., 2014, 2017; Gao et al., 2019). So, it is always good to go for the multi-component fittings of the galaxies. The robustness of the multi-component fittings over the two-component fittings has been discussed in Salo et al. (2015) for barred and unbarred galaxies. The two-component fittings of barred galaxies over estimate the mass, size and Sérsic index of the bulges but there is no effect on unbarred galaxies. In this subsection, we will use S^4G data to investigate the effect of multi-component fittings on our results. In the Fig. 6.7, we have shown the histograms of Sérsic index (top panel) and bulge ellipticity (bottom panel) for 27 galaxies, common in both survey, which are fitted by Sérsic profile. The red and blue filled histograms show the galaxies from SDSS and S^4G survey respectively and vertical lines represent the medians of respective data. We removed all the galaxies having bar and/or point source while making these histograms. The median of Sérsic indices is 1.02 (standard deviation = 1.16) for S^4G galaxies and 1.28 (standard deviation =0.95) for SDSS galaxies. On the other hand, the median of bulge ellipticity for S^4G is 0.41 (standard deviation = 0.21) and for SDSS is 0.31 (standard deviation =0.19). We can clearly see the clues of over-estimation of Sérsic index and under estimation of bulge ellipticity in SDSS data. In the estimation of structure parameters of galaxies, GALFIT is better than the GIM2D (Häussler et al., 2007). Hence, we can say that our result will not be affected by the multi-component fitting rather it will stand in the support of our results with much better confidence that the fraction of pseudo-bulges is increasing as Universe is getting older and older.

We have also calculated the distribution of the bulges with distance for S^4G survey after removing galaxies having bar and/or point source. Fig. 6.8 shows the distribution of classical and pseudo-bulges in the local Universe for S^4G survey. Top, middle, and bottom rows of this figure represent all, disc dominating, and bulge dominating galaxies respectively. Red and green curves represent classical bulges and pseudo-bulges respectively. Here, we have shown only Sérsic classification of the bulges because all three classifications give over all similar qualitative trends with small quantitative difference. From top two panels of the figure, one can easily notice that the pseudo-bulges dominates over classical bulges over whole range of distance. This domination is similar to the SDSS data in local volume. On comparison in very local volume, it seems that S^4G data show larger fraction of pseudo-bulges than the SDSS data. This is because all the points in S^4G data are equivalent to one point in SDSS data of the Fig. 6.2. If we take average of all the S^4G points, it will give comparable fraction as that in SDSS data. For example, average fraction of pseudo-bulges in all S^4G data is 0.66, which is very close to the fraction of pseudo-bulges in all SDSS data within statistical uncertainties.

In the bulge dominating galaxies (bottom panel of Fig. 6.8), the fraction of pseudo-bulges is less than the fraction of classical bulges i.e classical bulges are dominating over pseudo-bulges.

Fig. 6.8 Fractional distribution of classical and pseudo-bulges with distance in the S^4G survey. Top, middle, and bottom rows represent all, disc dominating, and bulge dominating galaxies respectively. Red and green curves represent classical bulges and pseudo-bulges respectively. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. Average fraction of the bulges here is equivalent to the local volume bulge fraction in SDSS data.

Fig. 6.9 Effect of disc inclination and telescope's resolution on classical bulges of model galaxies. Left and right columns show high-resolution and low-resolution galaxies respectively. Top and bottom rows represent edge-on and face-on fittings of the model galaxies respectively. The blue upper triangle solid curves represent the models $B/T = 0.5$ respectively and dots are just connecting points.

This is opposite to that in upper two panels but, it is equivalent to the fraction of bulges in bulge dominating galaxies of SDSS data. If we look at top and middle panels of Fig. 6.8, we can see that both of them are showing more or less similar fractions of bulges. It means that the over all fractional distribution of the bulges in S^4G data is governed by the disc dominating galaxies not by the bulge dominating galaxies. We found that the data which is fitted with Sérsic profile has more than 78% disc dominating galaxies. Since the S^4G survey is limited by the size and magnitude of the galaxies (Sheth et al., 2010). Therefore the fractional distribution of the bulges in the bottom panel is biased by the survey limitations. We are most probably seeing the merger dominated massive galaxies.

6.4 Effect of Disc Inclination and Instrument's Resolution

The state of art cosmological simulations and observational studies suggest that the Universe was dominated by mergers in the early epoch of galaxy formation (White & Rees, 1978b; Springel et al., 2006; Sinha & Holley-Bockelmann, 2012b). The violent relaxation of merged galaxies usually lead to the formation of classical bulges (Kauffmann et al., 1993b; Hopkins

et al., 2009c; Naab et al., 2014). Once formed, it is not an easy task to transform a classical bulge into a pseudo-bulge (Kumar et al., 2021) unless there is a central instability (e.g. bar) which can make it dynamically cool (Saha et al., 2012a, 2016a) by transferring significant angular momentum (Kataria & Das, 2019). But, instead of being dominated by classical bulges, the local Universe is dominated by the pseudo-bulges. Where have classical bulges gone in the evolution of galaxies? Is there any role of the two-component fitting? How do the disc surface density and inclination affect the two-component fitting? In the search of these questions as well as to test the reliability of Sérsic index method, we have generated $N-$body disc galaxies with classical bulges and performed two-dimensional bulge-disc decomposition using GALFIT.

Fig. 6.9 shows the effect of disc inclination (rows) and spatial resolution (columns) on the Sérsic index of classical bulges present in discs with a range of surface densities or in a range of R_e/R_{hlr}. In this figure, blue upper triangle solid curves represent the galaxy model with $B/T = 0.5$. We have conducted a similar analysis for several values of B/T. However, we show only one for brevity, given our focus on the importance of disc surface density with respect to the central bulge. In this section, we show that only the Sérsic index is not a reliable quantity to separate two classes of the bulges. We emphasize these points because it is one of the two parameters used in our new Sérsic-Kormendy classification of bulges. At a given telescope's resolution, the Sérsic index in edge-on fittings is always greater than the Sérsic index in face-on fittings. The edge-on decomposition of the model galaxies provides the same Sérsic index as reported by Dehnen (1993) for pure Hernquist profile. The reasons for this difference between face-on and edge-on fitting are (1) Hernquist density profile which is not truncated at large radius in our models and (2) the real size of bulges can be traced only in edge-on projection of galaxies. Hence, the Sérsic indices of the classical bulges depend on the disc inclination.

Each curve in Fig. 6.9 represents the dependence of the Sérsic index of a given classical bulge on the size (or surface density) of the hosting disc. The Sérsic index of edge-on galaxies increases with increasing R_e/R_{hlr} or we can say that the Sérsic index of edge-on galaxies increases with decreasing size of hosting disc because all the bulges have same scale radius in our models. But the Sérsic index of face-on galaxies first decreases and then increases with increasing R_e/R_{hlr}. In other words, one can think that the classical bulges easily pop-up in the low surface density discs. Similar results have been noticed in Fig. 6.3, where we saw decreasing classical bulge fraction with increasing R_e/R_{hlr} and then again increasing. The spatial resolution of the telescope also plays a crucial role in the determination of Sérsic index. The low-resolution image usually takes large number of stars in each pixel which increases central density of the galaxy. This increased central density mainly contributes to the bulge in the term of increasing Sérsic index. Therefore, the Sérsic index in low-resolution fittings is always greater than the high-resolution fittings. These results have direct significance on the

Fig. 6.10 Fractional distribution of classical and pseudo-bulges with r-band apparent magnitude ($m_{r,g}$) and stellar mass ($M_{*,g}$) of all the galaxies in the SDSS survey. Left, middle, and right columns show Sérsic, Kormendy and Sérsic-Kormendy classifications respectively. Red and green curves represent classical bulges and pseudo-bulges respectively. The numbers of classical bulge and pseudo-bulge galaxies at any point are shown with their respective colors. The fraction of pseudo-bulges increase in faint and low mass galaxies.

fraction of the bulges at high-redshift where the spatial resolution of the telescope decreases by a factor of distance. In Fig. 6.2, Some of the classical bulges at high-redshift could be the result of decreasing spatial resolution of the telescope.

6.5   Effect of data selection criteria

The results, we have presented for SDSS data, are based on a sample of galaxies which are best represented by two components with least uncertainty. Imposing this strict constraint on the parent data reduces the galaxies from faint and low-mass end of the distribution, see Fig. 6.1. Now, one can ask what will be the effect of this bias on our results? To understand the effect of reducing faint and low-mass galaxies from parent sample, we calculated the distribution of classical bulges and pseudo-bulges as a function of apparent magnitude and stellar mass of galaxies. It is shown in Fig. 6.10. The left, middle, and right columns of this figure show

the distribution of bulges based on Sérsic, Kormendy, and Sérsic-Kormendy classifications respectively. Top row shows the fraction of bulges with r-band apparent magnitude of galaxies, whereas bottom row displays fractional distribution of bulges with stellar mass of the galaxies. The number of classical bulge and pseudo-bulge galaxies at any point of magnitude and stellar mass are also shown with respective colors.

From Fig. 6.10, it is clear that the fraction of the pseudo-bulges increases when we move towards the faint and low-mass end of galaxies in all the classification schemes. On the other hand, the fraction of classical bulges increases when we go from faint and low-mass end to bright and high-mass end of the galaxies. If we had not imposed our strict selection criteria, we would have seen increase in pseudo-bulge fraction as compare to that reported in Section 6.3.1. We have verified it for parent sample but, for the brevity, is not shown here. Also, the inclusion of faint and low-mass galaxies will not affect our results discussed in Section 6.3.3 as those results are based on the mean of the bulge ellipticity of statistically significant number of galaxies. Finally, we would like to emphasize that including large number of galaxies will not affect our results though it will only reduce the statistical uncertainties of our results.

6.6   Discussion

In previous section, we have presented our results on the evolution of the bulges since $z = 0.1$ redshift. The effect of instrument's spatial resolution and galaxy inclination is shown using Milky Way type model galaxies. The correlation of the bulges with various photometric parameters and effect of multi-component fitting of the galaxy is also discussed. In this section, we interpret our results on the evolution of the bulges in observations.

The classification of bulges in classical bulge and pseudo-bulge categories has always been the topic of debate. Different people use different dividing criteria which suite their needs e.g $n = 2$ Sérsic index (Fisher, 2006b; Fisher & Drory, 2008b), Kormendy relation (Gadotti, 2009b), Color index (Lackner & Gunn, 2012), velocity dispersion (Fabricius et al., 2012), central stellar mass density (Cheung et al., 2012), etc. We defined bulges into classical and pseudo categories using a combination of Sérsic index and Kormendy relation which we call Sérsic-Kormendy classification and compared properties of the bulges in three classification schemes. We found that the results based on Sérsic classification and Kormendy classification deviate more from each other with increasing bulge to disc semi-major axis ratio. Results from combined Sérsic-Kormendy classification lies in between those from two individual methods.

The fraction of classical bulges and pseudo-bulges varies with the evolution of the Universe. As we go from low-redshift to high-redshift, the fraction of pseudo-bulges decreases while the fraction of classical bulge increases. At redshift $z \approx 0.016$ for Sérisc-Kormendy bulge

classification, both types of bulges contribute equally. Pseudo-bulges are thought to be formed due to secular evolution of galaxies. The ubiquitous distribution of pseudo-bulge hosting disc galaxies in the local volume raise the question at hierarchical nature of the Universe (Weinzirl et al., 2009b; Kormendy et al., 2010a; Fisher & Drory, 2011b). Since the existence of zoom-in and hydrodynamic simulations, many studies have shown the importance of gas against the destruction (or heating) of the disc in merger events. These studies show that the pseudo-bulge can also form in mergers in contrary to the secular evolution (Governato et al., 2009; Hopkins et al., 2009a; Moster et al., 2010; Guedes et al., 2013; Okamoto, 2013). Other studies suggest to improve the initial condition of galaxy formation (Peebles, 2020b). A proper understanding of the physical processes and/or the adjustment to the current cosmological model is still needed for better representation of the observable Universe.

We found that a large fraction of the pseudo-bulges are rarer in density as compare to the classical bulges relative to their hosting discs. Gadotti (2009b) also found that the pseudo bulges less are concentrated than the classical bulges at given bulge to disc mass ratio. High stellar density discs usually go through instabilities e.g. bars. Lütticke et al. (2000) studied a sample of edge-on galaxies from RC3 catalogue (Third Reference Catalogue of Bright Galaxies) and found that 45% of all bulges are boxy/peanut in shape and explained by the presence of bar. Rare density of pseudo-bulges is in agreement with previous studies where (Erwin & Debattista, 2017) claims that the semi-major axes of boxy/peanut shape bulges range from one-quarter to three-quarters of the full bar size. If the classical bulges are small in mass, these instabilities can hide them in (or sometime erode completely) and can result in composite bulges as seen in some observations (Erwin et al., 2015; Blaña Díaz et al., 2018; Erwin et al., 2021). The increasing fraction of pseudo-bulges with decreasing redshift could be the result of these hidden low mass classical bulges (Saha, 2015). Further, our result shows the bulge dominated systems also have higher fraction of pseudo bulges at low-redshift which is opposite to earlier claims Gadotti (2009b). Though our results supports the composite bulge scenario. A detailed multi-component decomposition of galaxies can reveal what fraction of the pseudo-bulges are composite bulges. Further these techniques can certainly help in improving our understanding of the galaxy evolution.

The two-dimensional photometric decomposition of the galaxies gives not only the quantitative information of the bulge and disc components but, it also provides the tool to understand the evolution of bulges. From the sub-sample of near face-on galaxies in SDSS data, we found that massive bulges are more round than the low mass bulges at all redshift. In local volume, classical bulges show mean ellipticity in the range 0.2 to 0.3, whereas pseudo-bulges show in the range 0.3 to 0.5 for Sérsic-Kormendy bulge classification which is consistent with previous studies of spiral galaxies (Fathi & Peletier, 2003; Méndez-Abreu et al., 2008). Also,

the mean ellipticity of both types of bulges grows with the increasing redshift. It indicates a clear morphological evolution of the bulges since $z = 0.1$. The decreasing mean ellipticity and increasing pseudo-bulge fraction point towards the secular evolution in the later time of Universe. Recent cosmological simulations have also shown that the rate of violent interactions (mergers) between galaxies decreases with the evolution of the universe and slowly, flyby interactions dominate the evolution (Sinha & Holley-Bockelmann, 2012b; An et al., 2019a).

The spatial resolution of the observing instrument also play a crucial role in the identification of the bulge type Fig.6.9. At low spatial resolution, a pseudo-bulge can be mistakenly classified as classical bulge in photometric decomposition of the bulge using Sérsic profile. Since the spatial resolution of the instrument decreases when we see a high-redshift object. Hence, the fraction of the pseudo-bulge at high-redshift could be higher than what we have calculated from the sample of photometric bulge-disc decomposition. A further investigation is needed to tightly constrain the fraction of the cold and hot fraction of stellar matter in the Universe at different redshift. Calculation of the exact fraction of ordered and random stellar orbits in disc galaxies at different redshift will help us in better understanding of the initial cosmological conditions and/or the underlying baryonic physics.

Our study is based on two-component photometric decomposition. The effect of multi-component decomposition of the galaxies is very crucial to understand the real statistics of the stellar matter distribution in various components of the galaxy. Two-component fitting usually gives higher Sérsic index and lower ellipticity in case of the barred galaxies (Figure 6.7). This can lead to small fraction of the pseudo-bulges and elongated bulges. Sometimes, photometric and kinematic decomposition of cold and hot components show contrary classification of the bulges (Gadotti, 2009b). For example, the excess light in the center of galaxy due to nuclear activity gives higher Sérsic index in photometric decomposition. The detailed relation in photometric and kinematic decomposition is necessary for better photometric classification criterion.

6.7   Summary

To understand the evolution of the bulges with the evolution of the Universe, we have used the archival data of two-dimensional bulge-disc decomposition of galaxies from SDSS DR7 (Simard et al., 2011). This data is constraint to nearly 40000 galaxies those can be well represented by two components. The classical bulges and pseudo-bulges are separated using $n = 2$ Sérsic index, Kormendy relation, and a combination of two. To explore the effect of multi-component fitting, we have also used the archival data of two-dimensional multi-component decomposition of local volume galaxies from S^4G survey (Salo et al., 2015). Further, we have

simulated Milky Way type model galaxies to investigate the effect of spatial resolution and disc inclination. The main findings of our analysis are as follows:-

(i) The fraction of pseudo-bulges dominates over classical bulges in the local Universe. As we go towards the high-redshift, the fraction of pseudo-bulges decreases smoothly. In the history of the Universe, there came a point (z $\approx$ 0.016 for Sérsic-Kormendy classification) when the fraction of classical bulges and pseudo-bulges was 50-50%. Universe is dominated by classical bulged galaxies before this point during its evolution while it is dominated by pseudo-bulges as soon it crosses this point.

(ii) In the local Universe, disc dominating galaxies show more pseudo-bulges as compare to bulge dominating galaxies. The fractional distributions of pseudo-bulges in Kormendy, and Sérsic-Kormendy classifications are quite similar. But, Sérsic classification shows lower pseudo-bulges than other two classification at low-redshift.

(iii) The fraction of the pseudo-bulges increases with increasing bulge to disc semi-major half-light ratio until $R_e/R_{hlr} \approx 0.6$. Compact and shorter bulges, as compared to their hosting discs, are usually classical in nature however, pseudo-bulges are diffuse and longer. In other words, concentrated discs harbour pseudo-bulges while rarer discs host classical bulges.

(iv) At large bulge to disc semi-major axis ratio, Kormendy classification shows more pseudo-bulges than other two classifications. This difference is more pronounced in disc dominating galaxies. Bulge dominating galaxies show very similar fraction in all classifications. For better division between classical and pseudo-bulges, we recommend to use a combination of Sérsic index and Kormendy relation based bulge classification.

(v) The mean ellipticity of pseudo-bulges is greater than the mean ellipticity of classical bulges in whole redshift range and it decreases with decreasing redshift indicating that the bulges are getting more axisymmetric with the evolution of galaxies. High-mass bulges are progressing towards the axisymmetry at more steep rate than the low-mass bulges.

(vi) In local Universe, nearly 75% of the visible matter is dominated by ordered rotational motion in disc galaxies. The existence of the rotation dominated Universe challenges the hierarchical nature of the Universe which suggests that most of the Universe should be dispersion dominated.

(vii) The evolution of the bulge does not have significantly strong correlation with the evolution of the photometric properties of galaxy e.g. bulge to total light ratio, disc inclination, bulge ellipticity, absolute magnitude of galaxy etc. Only absolute magnitude of galaxies and ellipticity of bulges show a negative correlation of $\approx$ 0.3 on the scale of unity in the low-redshift Universe that also drops in very local Universe.

Fig. 6.11 Sizes of the bulges and discs in our sample. Left and middle columns demonstrate absolute sizes of discs and bulges respectively as a function of redshift, whereas right column shows their comparison. Top and bottom rows display apparent sizes and physical sizes respectively.

(viii) Our results are consistent with the local volume survey S^4G. The multi-component fitting of the galaxies does not fade our conclusions. It stands with strong support of our results discussed in this article.

The tight constraint on the fraction of the classical bulges and pseudo-bulges with the redshift can be verified with next generation telescopes e.g. JWST, TMT, SKA etc. These telescope will provide unprecedented high-resolution view of the universe. Using the data obtained with these next generation telescope, we can better understand the underlying mechanism which played major role in the stability of discs during the growth and evolution of the structures.

6.8 Appendix: Comparison of Bulges and Discs

In Fig.6.11, we have shown the apparent and physical sizes of the bulges and discs as a function of redshift. Top and bottom rows show apparent sizes and physical sizes respectively. Left and middle columns display absolute sizes of discs and bulges respectively, whereas right column shows their comparison. There are some galaxies that fall out of the shown range of the axes. However, for better visualization, range of the axes are chosen arbitrarily. As expected, apparent

Fig. 6.12 Distribution of the bulge to disc central brightness ratio for the sample selected from SDSS survey with the bulge to disc semi-major half-light radius. The red curve shows the median of the distribution and shaded region represents $\pm 25\%$ deviation from median.

sizes of the bulges and discs decrease with increasing redshift, while physical sizes increase with increasing redshift. Comparison of bulges and discs show notable positive correlation. On average, larger discs host larger bulges and smaller discs hot smaller bulges. This correlation shows increasing dispersion with increasing disc size.

6.9   Appendix: Bulge to Disc Central Brightness

In Fig. 6.12, we have shown the ratio of bulge central brightness ($\Sigma_{(0,b)}$) to the disc central brightness ($\Sigma_{(0,d)}$) as a function of the bulge to disc semi-major half-light radius ratio. This ratio is calculated using the following expression,

$$\frac{\Sigma_{(0,b)}}{\Sigma_{(0,d)}} = \frac{B}{D} \times \frac{R_{hlr}^2}{R_e^2} \tag{6.6}$$

where, $\frac{B}{D}$ is the photometric bulge to disc light ratio obtained from two-component decomposition of the galaxies. R_e and R_{hlr} are the semi-major half-light radii of the bulge and disc components respectively. This figure is clearly showing that the bulge concentration is decreasing with increasing bulge to disc semi-major half-light radius ratio. It is evident that the R_e/R_{hlr} can be used as an indicator to measure the bulge concentration relative to the hosting disc.

7

Bulgeless Galaxies in the Illustris TNG50 Simulations: A Test for Angular Momentum Problem

7.1 Introduction

The Hubble classification of galaxies (Hubble, 1926) separates galaxies into different morphological classes according to the relative importance of the dispersion and rotation components of stars (also see, de Vaucouleurs, 1959c,b; van den Bergh, 2007). Spiral galaxies are the rotationally dominated systems with varying fractions of dispersion components at their centers, commonly known as bulges. In the photometric decomposition of the galaxies, the excess light above the exponential disk profile is defined as the bulge (see the book, Laurikainen et al., 2016). Bulges show a variety of surface brightness profiles displaying steep to flat variation with radius (Fisher, 2006c; Fisher & Drory, 2008c,e; Kumar et al., 2021; Kumar & Kataria, 2022). Bulges with steep and flat surface brightness profiles are known as classical bulges and pseudo-bulges respectively (Kormendy, 1993b; Andredakis & Sanders, 1994b; Kormendy et al., 2006b; Drory & Fisher, 2007b).

Several theoretical and numerical studies have shown that the merger of two galaxies results in the formation of classical bulges (Kauffmann et al., 1993b; Baugh et al., 1996; Aguerri et al., 2001c; Bournaud et al., 2005b; Hopkins et al., 2009b; Naab et al., 2014). The kinematic and morphological similarities of classical bulges and elliptical galaxies further support the fact that classical bulges are formed in violent relaxation processes (Bournaud et al., 2007b; Naab et al., 2014). On the other hand, pseudo-bulges are thought to form during the secular evolution of galaxies (Kormendy, 1993b; Heller & Shlosman, 1994b; Kormendy & Kennicutt, 2004c; Laurikainen et al., 2009). Classical bulges are usually redder than the pseudo-bulges in color. This is because their star formation histories differ significantly. Using observations of nearby galaxies, Fisher (2006e) concluded that the pseudo-bulges show long lasting star formation, whereas classical bulges show episodic star formation. Galaxies with pseudo-bulges also exhibit the presence of rings, bars and spirals in their nuclear regions (Andredakis & Sanders, 1994b; Carollo et al., 1997; Erwin & Sparke, 2002; Fisher, 2006e; Fisher & Drory, 2008c).

A significant fraction of spiral galaxies have been observed to have no bulges or very small bulges. This class of spiral galaxies are referred to as pure disk or flat galaxies or bulgeless galaxies (Karachentsev et al., 1993, 1999). Nearly $15 - 30\%$ of disk galaxies in the local volume are bulgeless (Kautsch et al., 2006; Barazza et al., 2008; Cameron et al., 2009; Karachentsev & Karachentseva, 2019). These galaxies show large amount of neutral and atomic hydrogen gas throughout their disks (Matthews & van Driel, 2000; Kaisin et al., 2020). However, they exhibit low global star formation rates because of low stellar surface densities and high stellar velocity dispersion (van der Hulst et al., 1993; Karachentseva et al., 2020; Banerjee et al., 2010).

It is well known that there is a tight correlation between the mass of supermassive black-holes (SMBHs) and the velocity dispersion (or luminosity) of bulges, which is in accordance to theories of hierarchical structure formation (Graham et al., 2001; Peng, 2007; Wang & Kauffmann, 2008). In the context of such studies, the presence of SMBHs and nuclear activities in bulgeless galaxies (Filippenko & Ho, 2003; Satyapal et al., 2016; Subramanian et al., 2016; Bohn et al., 2022) cannot be explained.

Bulges galaxies cannot be explained in the context of a merger dominated Universe. The existence of bulgeless galaxies is thus a challenge to the standard ΛCDM model of galaxy formation (Kormendy et al., 2010b; D'Onghia & Burkert, 2004). In the ΛCDM cosmological framework, galaxies form in the hierarchical manner. Several small structures merge over the course of evolution to form the large structures we see today (White & Rees, 1978b; Aguerri et al., 2001c; Bournaud et al., 2005b; Baugh, 2006). Numerically, it has been shown that the major mergers (collision of nearly equal mass galaxies) disrupt the disks and lead to the formation of elliptical galaxies (Barnes, 1992; Cox & Loeb, 2008). Though the minor mergers

(collision of significantly unequal mass galaxies) do not disrupt the disks, they can heat the disks dynamically to form the massive bulges (Hopkins et al., 2010b; Das et al., 2012; Kumar et al., 2021). Therefore, mergers are expected to form spheroid like structures and not pure disks.

Recent advancements in numerical simulations have shown that the feedback-driven outflows of gas in dwarf galaxies can prevent the formation of classical bulges (D'Onghia & Burkert, 2004; Governato et al., 2010; Robertson et al., 2004). These simulations involve exceptionally intense feedback-driven gas removal from the center of the low-mass galaxies to produce bulgeless systems. Furthermore, simulated bulgeless galaxies are centrally concentrated, small in size, and have lower angular momentum than their observational counterparts (D'Onghia & Burkert, 2004; Piontek & Steinmetz, 2011). In cosmology, this controversy is known as the angular momentum problem or angular momentum catastrophe (van den Bosch et al., 2001; D'Onghia & Burkert, 2004). The rotation curves of bulgeless galaxies reveal that most of them are dark matter dominated systems. Dark matter is thought to stabilize the disk against external perturbations, like the classical bulge does in galaxy disks (Samland & Gerhard, 2003).

There is plenty of literature available on the formation and stability of bulgeless galaxies in isolation and in the non-cosmological context. But their study in the cosmological environment is still not understood. In this chapter, we investigate the formation of bulgeless galaxies in the cosmological setting using the state-of-art cosmological simulations, Illustis TNG. We aim to test how successful the TNG galaxy formation model is in reproducing bulgeless galaxies and whether they are comparable to those observed. Using the Illustris TNG data products we have estimated the fraction of bulgeless galaxies, their dark matter content, angular momentum, and merger histories.

We have arranged the chapter as follows: In Section 7.2, we have briefly described the Illustris TNG suite of cosmological simulations. We have talked about the automated pipeline for photometric decomposition of simulated galaxies in Section 7.3. Identification of the bulgeless galaxies is described in Section 7.4 along with the control sample for comparison.

7.2 The Illustris TNG simulations

To study the formation of the bulgeless galaxies in cosmological context, we have used the sample of galaxies from the IllustrisTNG project[1]. The IllustrisTNG project, TNG in short, is 'the next generation' follow-up of the Illustris project (Genel et al., 2014; Vogelsberger et al., 2014b,a; Nelson et al., 2015; Sijacki et al., 2015). TNG is a suite of gravo-

---

[1]https://www.tng-project.org/

magnetohydrodynamical, large scale, cosmological simulations (Nelson et al., 2019a). It comprises three cosmological volumes, namely TNG50, TNG100, and TNG300 having box sizes approximately 50, 100, and 300 Mpc respectively. For each cosmological volume, there have been performed low-resolution and dark matter only runs as well. The three boxes enable the study of different aspects of structure formation. TNG300 is useful in the investigation of large scale structures (e.g., clustering of the galaxies), and TNG50 is capable in the understanding of small scale properties (e.g., structure of the galaxies). On the other hand, TNG100 acts as a bridge between TNG300 and TNG50 for detailed study of structure formation. TNG100 and TNG300 have been introduced in Marinacci et al. (2018); Naiman et al. (2018); Nelson et al. (2018); Pillepich et al. (2018b); Springel et al. (2018). TNG50 is discussed in Nelson et al. (2019a,b); Pillepich et al. (2019). Now, all three runs are publicly available in their entirety (Nelson et al., 2019a). The data release includes snapshots, halo and subhalo catalogs, and merger trees for each run.

All the TNG simulations are performed with the moving-mesh and massively parallel code AREPO (Springel, 2010) starting from the redshit z=127 to z=0. The initial conditions for the simulations are $\Omega_{\Lambda,0} = 0.6911$, $\Omega_{m,0} = 0.3089$, $\Omega_{b,0} = 0.0486$, $n_s = 0.9667$, $\sigma_8 = 0.8159$, and $h = 0.6774$ that are consistent with Planck Collaboration et al. (2016). TNG galaxy formation model includes physics of gas cooling, star formation, stellar evolution, metal enrichment, stellar feedback, supermassive black-hole growth, supermassive black-hole feedback, magnetic field, etc (Vogelsberger et al., 2013; Weinberger et al., 2017; Pillepich et al., 2018a). TNG has successfully reproduced several observational results in different regimes. For example, $g - r$ color bimodality in SDSS galaxies (Nelson et al., 2018), galaxy correlation function in SDSS (Springel et al., 2018), [Eu/Fe] spread at low metallicities in the Milky Way (Naiman et al., 2018), mass-size relation of late- and early-type galaxies (Genel et al., 2018), mass-metallicity relation of gas (Torrey et al., 2019), dark matter mass and circular velocity of the Milky Way mass galaxies (Lovell et al., 2018), morphology of Pan-STARRS galaxies (Rodriguez-Gomez et al., 2019), etc.

7.3 Pipeline for morphological decomposition

In this study, we have selected sample galaxies from TNG50-1 (hereafter TNG50) run because this is the highest resolution run with full physics available in IllustrisTNG suite of simulations. In IllustrisTNG, galaxies are the subhalos identified by the SUBFIND alogrithm (Springel et al., 2001b). We have used the available catalog of the subhalos and restricted ourselves to the galaxies having stellar masses greater than $10^9 M_\odot$. This choice removes the dwarf galaxies and provides the sample galaxies having significant number of stellar particles ($> 2.2 \times 10^4$)

that is necessary for the reliable study of the galactic components. The mass constraint on the catalog returns a total of 2512 sample galaxies.

For the morphological decomposition of these galaxies, we have used the up-to-date version of the two-dimensional photometric decomposition tool, GALFIT (Peng et al., 2002b, 2010b). GALFIT is a C language based least-square fitting code that uses Levenberg–Marquardt algorithm for searching the optimal parameters to the fitting. It is capable of fitting multiple components simultaneous. GALFIT takes the image of the galaxy in fits format and initial guess parameters as input for the fitting. To perform the multi-component fitting on such a large sample of the galaxies, we have developed an automated python language based pipeline. It acts as a wrapper for the GALFIT code. This pipeline reads the TNG galaxies for given list of subhalo IDs, estimates the center of mass for stellar component, calculates the total stellar angular momentum within the radius containing 90% of the stellar mass, rotates all the particles so that the total angular momentum within radius containing 90% of stellar mass aligns in $z-$direction of the Cartesian co-ordinates, makes fits format image of galaxy in face-on view, finds initial guess for the fitting profile, generates input file for GALFIT, executes GALFIT, and stores the output in a txt file.

Our complete pipeline consists of 3 separate scripts. The first script of the pipeline calculates the center of mass (CM) for the galaxy. Then it estimates the radius containing 90% of the stellar mass (r_{90}) and measures total angular momentum within this radius ($\vec{L}_{90}$). Both the quantities along with the IDs of the galaxies are stored in a text file. The second script of the pipeline uses the parameters estimated in the first script to rotate the galaxy in such a way that $\vec{L}_{90}$ aligns in $z-$direction of the Cartesian co-ordinates. Then it measures the complex amplitude of m=2 Fourier mode in x-y plane using the following equation,

$$A_m(<R) = \sum_j m_j e^{-im\phi_j}, \tag{7.1}$$

where A_m is the complex amplitude of mth Fourier mode within radius R, m_j is the mass of jth particle, ϕ_j is the azimuth angle of jth particle, and i is the iota (see e.g., Kumar et al., 2021, 2022b). The normalized amplitude of m=2 Fourier mode ($|A_2|/|A_0|$), phase angle ($\tan^{-1}[\frac{Im(A_2)}{Re(A_2)}]$), and radius along with IDs of galaxies are saved in a text file. The third script of the pipeline reads the output from first script to save a fits image in face-on projection. It also makes use of the outputs from second script to guess the initial parameters of the components to be fitted. For example, if one wants to fit the central component of the galaxy, it will take the phase angle of m=2 Fourier mode as a guess for the position angle.

GALFIT provides several in-built profiles for the fitting. Currently, we have included only Sérsic, Exponential disk, Edge-on disk, and Ferrer profiles in our pipeline. The Sérsic profile

for the two-dimensional decomposition is expressed as,

$$\Sigma(R) = \Sigma_e \exp\left[-b_n\left\{\left(\frac{R}{R_e}\right)^{\frac{1}{n}} - 1\right\}\right] \tag{7.2}$$

where R_e is the effective radius, Σ_e is the surface density at the effective radius, and b_n is a function of Sérsic index n. The exponential disk profile for face-on decomposition is given by,

$$\Sigma_d(R) = \Sigma_{d0} \exp\left(-\frac{R}{R_s}\right) \tag{7.3}$$

where Σ_{d0} is the surface density at center of the disk and R_s is scale radius of the disk. The edge-on disk profile for edge-on decomposition is modeled by

$$\Sigma_d(R, z) = \Sigma_{d0}\left(\frac{R}{R_s}\right) K_1\left(\frac{R}{R_s}\right) \text{sech}^2\left(\frac{z}{z_0}\right) \tag{7.4}$$

where K_1 is the Bessel function and z_0 is the scale height of the disk.

While performing 2D photometric decomposition, we have centred face-on galaxy in a box of size $3r_{90}$ with grid (or pixel) size equal to the gravitational softening (ε) of the snapshot. The choice of box size (i.e. $3r_{90}$) is arbitrary. The only motivation for this choice is to consider most of the stellar mass during the fitting.

Previous studies of galaxy bulges suggest that the Sérsic index $n = 2$ is a good proxy for distinguishing between classical bulges and pseudobulges (Fisher, 2006c; Fisher & Drory, 2008c). Generally, classical bulges have a Sérsic index $n > 2$ while pseudobulges have a Sérsic index $n < 2$. We have applied this classification scheme for bulges in our study.

7.4   Sample of bulgeless galaxies and galaxies with bulge

The sample of bulgeless galaxies is selected using the single Sérsic profile fitting as done in Grossi et al. (2018) for observational galaxies. First, we performed single Sérsic component fitting on the whole sample of TNG50 galaxies with our pipeline. Then, we selected all those galaxies which are best fitted with single Sérsic component, and have Sérsic index less than 1.5. We removed all the unresolved galaxies that have effective radius smaller than three softening length of the simulation. All these constraints return a sample of 16 unbarred bulgeless galaxies in TNG50 run.

Next, we execute our pipeline with two Sérsic components and selected those galaxies that are best fitted with two Sérsic profiles. Then we removed all the galaxies that were best fitted

Fig. 7.1 Visual comparison of observed bulgeless galaxies with those in TNG50 simulation. Top row show three observed bulgeless galaxies and bottom row display bulgeless galaxies formed in TNG50 simulation. Note that in TNG50 images, we have only shown stellar component.

with single Sérsic profile. From this sample, we separated disk and bulge/bar components using Sérsic index, effective radius, and normalized amplitude of m=2 Fourier mode. The component with Sérsic index less than 1.5 and the largest effective radius is considered disk and the other component is considered bulge/bar. If the normalized amplitude of m=2 Fourier mode is less than 0.1, we call the smaller component bulge; otherwise, we call it bar. Bulge with Sérsic index less than 2 are pseudo-bulge and bulge with Sérsic index greater than 2 are classical bulge. Further, we removed galaxies that have bulge/bar effective radius smaller than two softening length of the simulation. Similar analogy is used for three component fitting.

In Fig. 7.1, we have shown example of bulgeless galaxies from observation and TNG50 simulation. Top row panels of the figure display observed examples of bulgeless galaxies drawn from Sloan Digital Sky Survey (SDSS) and bottom row panels show the simulated examples of bulgeless galaxies taken from TNG50 data. These are pure disk galaxies. One can barely notice the central bulge component in these galaxies. Note that in TNG50 images, we have only shown stellar component.

Fig. 7.2 Number distribution of different classes of galaxies in TNG50 simulation. These galaxies are identified using our two dimensional multi-component fitting pipeline.

7.5   Results

7.5.1   fraction of bulgeless galaxies

Local volume observation of spiral galaxies shows that approximately $15 - 30\%$ of disk galaxies are bulgeless (Kautsch et al., 2006; Barazza et al., 2008; Cameron et al., 2009; Karachentsev & Karachentseva, 2019). The fraction of barred bulgeless galaxies is around 87% of all bulgeless galaxies (Barazza et al., 2008). To compare the statistics of bulgeless galaxies in observations and TNG50 simulation, we took all the galaxies which are best fitted by disk and bulge or/and bar components. We found that 25% of TNG50 disk galaxies are bulgeless. The fraction of barred bulgeless galaxies is 81% of bulgeless galaxies. The fraction of bulgeless galaxies in TNG50 is reasonably good match with the observed fraction in local volume. In Fig. 7.2, we have shown different classes of galaxies and their numbers found in the TNG50 simulation.

7.5.2   Specific angular momentum

Observations have shown that bulgeless galaxies are rich in specific angular momentum (van den Bosch et al., 2001; D'Onghia & Burkert, 2004). To test whether bulgeless galaxies in TNG50 are also rich in angular momentum or not, we have calculated their specific angular

Fig. 7.3 baryon to dark matter specific angular momentum ratio distribution of bulgeless galaxies and galaxies with bulge. A significant fraction of bulgeless galaxies shows higher specific angular momentum as compare to the galaxies with bulge.

momenta within two half stellar mass radius [2]. In Fig. 7.3, we have shown the ratio of baryonic and dark matter angular momenta for bulgeless galaxies and galaxies with a bulge. The distribution for bulgeless galaxies and galaxies with bulges are respectively coded with orange and blue colours. It is evident that the bulgeless galaxies are rich in specific angular momentum. The majority of galaxies with bulge are distributed in the low specific angular momentum region.

We have also estimated specific angular momenta of stellar and gas components separately and computed their ratios with specific angular momentum of dark matter. Similar to the baryon to dark matter specific angular momentum ratio, both, gas to dark matter and stellar to dark matter specific angular momentum ratios exhibit distinct distribution for bulgeless galaxies and galaxies with bulges. For brevity, we have not included these results in this chapter.

7.5.3 Baryonic to dark matter mass ratio

Bulgeless galaxies are generally dark matter dominated systems (Samland & Gerhard, 2003). The presence of dark matter stabilizes the disk against external disturbances. Here, we have calculated baryonic to dark matter mass ratio with two half stellar mass radius for bulgeless

---

[2]Here, we define specific angular momentum as $l = L/M$, where $L = \sqrt{(\sum_i L_{x,i})^2 + (\sum_i L_{y,i})^2 + (\sum_i L_{z,i})^2}$, $M = \sum_i m_i$, and i runs over all the particles within a fixed radius.

Fig. 7.4 baryon to dark matter mass ratio distribution of bulgeless galaxies and galaxies with bulge. Most of the bulgeless galaxies lies in $0 < M_{bary}/M_{dm} < 1$ range and galaxies with falls in wider range. It means bulgeless galaxies are dark matter dominating as compare to galaxies with bulge.

galaxies and for the control sample of galaxies with bulge. Fig. 7.4 represents baryonic to dark matter mass ratio distribution of bulgeless galaxies and galaxies with bulge in TNG50 simulation. Unlike stellar metallcity and specific angular momentum, baryonic to dark matter mass ratio of bulgeless galaxies is not very different from galaxies with bulges. Both the distributions peaks in $0 < M_{bary}/M_{dm} < 1$ range. However, concentrated distribution of bulgeless galaxies and wider distribution of galaxies with bulges indicate that bulgeless galaxies are dark matter dominated.

7.5.4 Stellar metallcity

In Fig. 7.5, we have shown the stellar metallicity distribution of bulgeless galaxies and compared it with the stellar metallcity of galaxies with bulge. Bulgeless galaxies and galaxies with bulges are respectively represented with orange and blue colours. Here, we have considered all the elements above helium (He) as metal and have shown mass weighted average stellar metallicity within two half stellar mass radius. It is clear that most of the bulgeless galaxies are metal-deficient as compared to galaxies with bulges. Galaxies with bulges show a wide range of metallicities, whereas bulgeless galaxies exhibit a small metallicity range. For the majority of bulgeless galaxies, stellar metallicity is less than the solar metallicity (solar metallicity ~ 0.02).

Fig. 7.5 Stellar metallicity distribution of bulgeless galaxies and galaxies with bulge. Bulgeless galaxies are metal-deficient and show narrow metallicity range. However, galaxies with bulge exhibit large metallicity range.

Observed bulgeless galaxies also typically exhibit low metallicity (Matthews & Gallagher, 1997).

7.6   Conclusions

We have investigated properties of bulgeless galaxies using Illustris TNG50 cosmological data products at redshift zero. To find the bulgeless galaxies, we have developed an automated python language based pipeline for the two dimensional photometric decomposition of galaxies. Our pipeline uses the GALFIT code at its core to decompose the simulated galaxies. The main findings of our study are the following.

- In TNG50, approximately 25% of disk galaxies are bulgeless. Out of the bulgeless galaxies around 81% are barred. These fractions of bulgeless and barred bulgeless galaxies are in agreement with the observed fraction of bulgeless galaxies in the local volume.

- Specific angular momentum of bulgeless galaxies in TNG50 is higher than the galaxies with bulges. The majority of bulgeless galaxies have baryon to dark matter specific

angular momentum ratio from 2 to 13. The gas component of bulgeless galaxies shows a higher specific angular momentum as compared to galaxies with bulges.

- We found that the bulgeless galaxies in TNG50 are dark matter dominated. However, galaxies with bulges have a wide range of baryon to dark matter fraction.

- Bulgeless galaxies in TNG50 show lower stellar metallicity values compared to control sample of galaxies with bulges. Most of the bulgeless galaxies show mass weighted average metallicities less than the solar metallicity value.

So to summarise we find that the bulgeless galaxies in TNG50 reasonably represent the characteristic properties of observed bulgeless galaxies. Thus, we conclude that the galaxy formation model of TNG50 is capable of producing bulgeless galaxies analogous to their observed counterpart.

"Everything dies, from the smallest blade of grass to the biggest galaxy."

– **Stephen R. Donaldson**

8

Concluding Summary and Future Directions

8.1 Summary

Previous chapters have discussed details of the individual work completed during the PhD period, including one on-going project. Here, we provide a brief summary of the work presented in this thesis.

In the Chapter 3, we have investigated the effect of minor flybys on the bulges, disks, and spiral arms of Milky Way mass galaxies for two types of bulges – classical bulges and boxy/peanut pseudo-bulges. Our N-body simulations comprise of two disk galaxies of mass ratios 10:1 and 5:1. We varied the distance of closest approach (pericenter distance) keeping the disks of the galaxies in their orbital plane. We performed photometric and kinematic bulge-disk decomposition at regular time steps and traced the evolution of the disk size, spiral structure, bulge Sérsic index, bulge mass, and bulge angular momentum. Our results show that the main effect on the disks is disk thickening, which is seen as the increase in the ratio of disk scale height to scale radius. The strength of the spiral structure A_2/A_0 shows small oscillations about the mean time-varying amplitude in the pseudo-bulge host galaxies. The flyby has no significant effect on the non-rotating classical bulges, which shows that these bulges are extremely stable in galaxy interactions. However, the pseudo-bulges become dynamically

hotter in flybys indicating that flybys may play an important role in accelerating the rate of secular evolution in disk galaxies. This effect on pseudo-bulges is a result of their rotational nature as part of the bar. Also, flybys do not affect the time and strength of bar buckling.

In the Chapter 4, we have tested whether the excitation of vertical breathing motion in a galaxy disk are directly linked to tidal interactions by constructing a set of N-body models (with mass ratio 5:1) of unbound, single fly-by interactions with varying orbital configurations. We first reproduce the well-known result that such fly-by interactions can excite strong transient spirals (lasting for $\sim 2.9 - 4.2\ Gyr$) in the outer disc of the host galaxy. The generation and strength of the spirals are shown to vary with the orbital parameters (the angle of interaction, and the orbital spin vector). Furthermore, we demonstrate that our fly-by models exhibit coherent breathing motion whose amplitude increases with height. The amplitudes of the breathing motion show characteristic modulation along the azimuthal direction, with compression breathing motions coinciding with the peaks of the spirals and expanding breathing motions falling in the inter-arm regions – a signature of a spiral-driven breathing motion. These breathing motions in our models end when the strong tidally-induced spiral arms fade away. Thus, it is the tidally-induced spirals which drives the large-scale breathing motions in our fly-by models, and the dynamical role of the tidal interaction in this context is indirect.

In the Chapter 5, we have shown that the dark matter halo shape affects bar formation and bar buckling. We have performed N-body simulations of bar buckling in non-spherical dark matter halos and traced bar evolution for 8 Gyr. We find that bar formation is delayed in oblate halos, resulting in delayed buckling whereas bars form earlier in prolate halos leading to earlier buckling. However, the duration of first buckling remains almost comparable. All the models show two buckling events but the most extreme prolate halo exhibits three distinct buckling features. Bars in prolate halos also show buckling signatures for the longest duration compared to spherical and oblate halos. Since ongoing buckling events are rarely observed, our study suggests that most barred galaxies may have more oblate or spherical halos rather than prolate halos. Our measurement of BPX structures also shows that prolate halos promote bar thickening and disc heating more than oblate and spherical halos.

In the Chapter 6, we have quantified the evolution of bulges since $z = 0.1$ using photometric parameters of nearly 39,000 unbarred disc galaxies from SDSS DR7 which are well represented by two components. We adopted a combination of the Sérsic index and Kormendy relation to separate classical bulges and disc-like pseudo-bulges. We found that the fraction of pseudo-bulges smoothly increases, and the fraction of classical bulges smoothly decreases as the Universe gets older. In the history of the Universe, there comes a point ($z \approx 0.016$) when classical bulges and pseudo-bulges become equal in number. The fraction of pseudo-bulges rises with increasing bulge to disc half-light radius ratio until $R_e/R_{hlr} \approx 0.6$ suggesting concentrated

disc is the most favourable place for pseudo-bulge formation. The mean ellipticity of pseudo-bulges is always greater than that of classical bulges and it decreases with decreasing redshift indicating that the bulges tend to be more axisymmetric with evolution. Also, the massive bulges are progressing towards axisymmetry at a steeper rate than the low-mass bulges. There is no tight correlation of bulge Sérsic index evolution with other photometric properties of the galaxies. Using the sample of multi-component fitting of S^4G data and $N-$body galaxy models, we have verified that our results are consistent and even more pronounced with multi-component fitting and high-resolution photometry.

In the Chapter 7, we have studied bulgeless galaxies in the Illustris TNG50 simulations to test its galaxy formation model. We selected all the redshift zero galaxies with stellar mass greater than $10^9 M_\odot$ and separated sample of bulgeless galaxies using two dimensional photometric decomposition. We calculated distributions of parameters for bulgeless galaxies and compared with a control sample of galaxies with bulge. We found that bulgeless galaxies in TNG50 exhibit low stellar metallicity, high specific angular momentum, and large dark matter fraction as compared to the galaxies with bulge. Nearly 25% of disk galaxies in TNG50 are bulgeless and 81% of these bulgeless galaxies are barred. Fraction of bulgeless galaxies in TNG50 matches well with observed fraction. We conclude that the TNG50 physics of galaxy formation is capable of producing observed characteristics of bulgeless galaxies in local volume.

8.2   Future Directions

The work presented in this thesis opens up various directions to explore in the future. Here, we list some possible projects to carry forward based on our individual work.

The work we presented in chapter 3 showed the effect of fly-by interactions on the evolution of bulge, disk, and spiral arms in a Milky Way mass galaxy. We performed purely $N-$body simulations of single minor fly-by interactions. For simplicity, we considered fly-by interactions on prograde orbits and varied the paricenter distances. Galaxies also contain a significant amount of gas, the fly-by interactions can occur between similar mass galaxies, the orbit of the interaction can be inclined, and a galaxy may experience more than one fly-by interaction simultaneously. These possibilities open the door for future projects where one can explore the consequences of these varying parameters on disk thickening, spiral arm formation, and growth of bulges. These possible projects will also include the evolution of galaxies in cosmological environments, such as groups and clusters of galaxies.

Using purely $N-$body simulation of fly-by interactions with various orbital configuration, we showed that the excitation of wave-like breathing motion in the Milky Way is related to its

spiral arms. The effect of tidal interactions is indirect. Our simulations do not account for the presence of gas and bar. The Milky Way hosts a strong bar at its center. One can study the contribution of bar on the wave-like breathing motion. The role of breathing motion on disk thickening and gas flaring will also be interesting to investigate. Studying breathing motion can be useful to understand the interaction history of the Milky Way and its siblings.

One of our studies showed that the dark matter distribution around the disk galaxies affects the formation of bars and the resultant boxy/peanut/x-shape (BPX) bulges. The shape of dark matter halo leaves its imprint on baryonic matter. A detailed study of BPX bulges in simulated galaxies may give us hints of dark matter distribution in observed disk galaxies. In this study, we did not include central bulge, super massive black hole (SMBH), and gas physics. It will be interesting to study the role of a central potential and gas physics on bar buckling and resultant BPX bulge formation in non-spherical dark matter distributions.

In our work on the cosmic evolution of disk-like pseudo-bulges, we have shown that the fraction of pseudo-bulges is increasing as the Universe becomes older. A large fraction of the local volume is rotation dominated but in the ΛCDM model of hierarchical structure formation, we expect a dispersion dominated Universe. Our results are thus useful for constraining the formation and evolution of bulges. These results may also be important for setting the initial conditions in cosmological simulations.

References

Aarseth S. J., Gott J. R. I., Turner E. L., 1979, *ApJ*, 228, 664

Abadi M. G., Navarro J. F., Steinmetz M., Eke V. R., 2003, *ApJ*, 597, 21

Abazajian et al 2009, *ApJ Suppl.*, 182, 543

Aguerri J. A. L., Balcells M., Peletier R. F., 2001a, *A & A*, 367, 428

Aguerri J. A. L., Balcells M., Peletier R. F., 2001b, *A & A*, 367, 428

Aguerri J. A. L., Balcells M., Peletier R. F., 2001c, *A & A*, 367, 428

Aguerri J. A. L., Elias-Rosa N., Corsini E. M., Muñoz-Tuñón C., 2005, *A & A*, 434, 109

Aguerri J. A. L., Méndez-Abreu J., Corsini E. M., 2009, *A & A*, 495, 491

An S.-H., Kim J., Moon J.-S., Yoon S.-J., 2019a, *ApJ*, 887, 59

An S.-H., Kim J., Moon J.-S., Yoon S.-J., 2019b, The Astrophysical Journal, 887, 59

Andredakis Y. C., Sanders R. H., 1994a, *MNRAS*, 267, 283

Andredakis Y. C., Sanders R. H., 1994b, *MNRAS*, 267, 283

Antoja T., et al., 2018a, *Nature*, 561, 360

Antoja T., et al., 2018b, *Nature*, 561, 360

Araki S., 1985, PhD thesis, MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

Arp H., 1966, *ApJ Suppl.*, 14, 1

Athanassoula E., 2002, *ApJL*, 569, L83

Athanassoula E., 2003, *MNRAS*, 341, 1179

Athanassoula E., 2005a, *MNRAS*, 358, 1477

Athanassoula E., 2005b, *MNRAS*, 358, 1477

Athanassoula E., 2008, in Bureau M., Athanassoula E., Barbuy B., eds, IAU Symposium Vol. 245, Formation and Evolution of Galaxy Bulges. pp 93–102, doi:10.1017/S1743921308017389

Athanassoula E., 2013, Bars and secular evolution in disk galaxies: Theoretical input. p. 305

Athanassoula E., 2016a, Boxy/Peanut/X Bulges, Barlenses and the Thick Part of Galactic Bars: What Are They and How Did They Form?. p. 391, doi:10.1007/978-3-319-19378-6_14

Athanassoula E., 2016b, Boxy/Peanut/X Bulges, Barlenses and the Thick Part of Galactic Bars: What Are They and How Did They Form?. p. 391, doi:10.1007/978-3-319-19378-6_14

Athanassoula E., Lambert J. C., Dehnen W., 2005, *MNRAS*, 363, 496

Athanassoula E., Machado R. E. G., Rodionov S. A., 2013, *MNRAS*, 429, 1949

Balland C., Silk J., Schaeffer R., 1998, *ApJ*, 497, 541

Banerjee A., Matthews L. D., Jog C. J., 2010, *Nature Astronomy*, 15, 89

Barazza F. D., Jogee S., Marinova I., 2008, *ApJ*, 675, 1194

Barnes J. E., 1992, *ApJ*, 393, 484

Barnes J. E., Hernquist L., 1992, *Anu Rev. in Astronomy and Astrophysics.*, 30, 705

Barnes J., Hut P., 1986, *Nature*, 324, 446

Baugh C. M., 2006, Reports on Progress in Physics, 69, 3101

Baugh C. M., Cole S., Frenk C. S., 1996, *MNRAS*, 283, 1361

Behroozi P. S., Wechsler R. H., Conroy C., 2013, *ApJ*, 770, 57

Bekki K., Shioya Y., 1997, *ApJL*, 478, L17

Bell E. F., McIntosh D. H., Katz N., Weinberg M. D., 2003, *ApJ Suppl.*, 149, 289

Berentzen I., Athanassoula E., Heller C. H., Fricke K. J., 2004, *MNRAS*, 347, 220

Berentzen I., Shlosman I., Jogee S., 2006, *ApJ*, 637, 582

Bertola F., Capaccioli M., 1975, *ApJ*, 200, 439

Bett P. E., Frenk C. S., 2012, *MNRAS*, 420, 3324

Bett P., Eke V., Frenk C. S., Jenkins A., Okamoto T., 2010, *MNRAS*, 404, 1137

Binney J., Tremaine S., 2008a, Galactic Dynamics: Second Edition

Binney J., Tremaine S., 2008b, Galactic Dynamics: Second Edition. Princeton University Press

Binney J., Gerhard O. E., Stark A. A., Bally J., Uchida K. I., 1991, *MNRAS*, 252, 210

Binney J., Gerhard O., Spergel D., 1997, *MNRAS*, 288, 365

Birnboim Y., Dekel A., 2003, *MNRAS*, 345, 349

Blaña Díaz M., et al., 2018, *MNRAS*, 481, 3210

Blitz L., Spergel D. N., 1991, *ApJ*, 379, 631

Bohn T., Canalizo G., Satyapal S., Sales L. V., 2022, *ApJ*, 931, 69

Bournaud F., Jog C. J., Combes F., 2005a, *A & A*, 437, 69

Bournaud F., Jog C. J., Combes F., 2005b, *A & A*, 437, 69

Bournaud F., Combes F., Jog C. J., Puerari I., 2005c, *A & A*, 438, 507

Bournaud F., Jog C. J., Combes F., 2007a, *A & A*, 476, 1179

Bournaud F., Jog C. J., Combes F., 2007b, *A & A*, 476, 1179

Bournaud F., Jog C. J., Combes F., 2007c, *A & A*, 476, 1179

Brooks A., Christensen C., 2016, in Laurikainen E., Peletier R., Gadotti D., eds, Astrophysics and Space Science Library Vol. 418, Galactic Bulges, Astrophysics and Space Science Library, Volume 418. ISBN 978-3-319-19377-9. Springer International Publishing Switzerland, 2016, p. 317. p. 317, doi:10.1007/978-3-319-19378-6_12

Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, *ApJ*, 555, 240

Bureau M., Athanassoula E., 2005, *ApJ*, 626, 159

Bureau M., Freeman K. C., 1999, *AJ*, 118, 126

Burkert A., Naab T., 2003, Major Mergers and the Origin of Elliptical Galaxies. pp 327–339, doi:10.1007/978-3-540-45040-5_27

Cabrera-Lavers A., González-Fernández C., Garzón F., Hammersley P. L., López-Corredoira M., 2008, *A & A*, 491, 781

Cameron E., Driver S. P., Graham A. W., Liske J., 2009, *ApJ*, 699, 105

Carlin J. L., et al., 2013, *ApJL*, 777, L5

Carollo C. M., Stiavelli M., de Zeeuw P. T., Mack J., 1997, *AJ*, 114, 2366

Carrillo I., et al., 2018, *MNRAS*, 475, 2679

Carrillo I., et al., 2019, *MNRAS*, 490, 797

Cervantes Sodi B., Sánchez García O., 2017, *ApJ*, 847, 37

Cervantes Sodi B., Li C., Park C., 2015, *ApJ*, 807, 111

Cheung E., et al., 2012, *ApJ*, 760, 131

Choi H., Yi S. K., 2017a, *ApJ*, 837, 68

Choi H., Yi S. K., 2017b, *ApJ*, 837, 68

Ciambur B. C., Graham A. W., Bland-Hawthorn J., 2017, *MNRAS*, 471, 3988

Ciotti L., Bertin G., 1999, *A & A*, 352, 447

Collier A., 2020, *MNRAS*, 492, 2241

Collier A., Shlosman I., Heller C., 2018, *MNRAS*, 476, 1331

Collier A., Shlosman I., Heller C., 2019, *MNRAS*, 488, 5788

Combes F., 2001a, in Aretxaga I., Kunth D., Mújica R., eds, Advanced Lectures on the Starburst-AGN. p. 223 (`arXiv:astro-ph/0010570`), doi:10.1142/9789812811318_0006

Combes F., 2001b, in Aretxaga I., Kunth D., Mújica R., eds, Advanced Lectures on the Starburst-AGN. p. 223 (`arXiv:astro-ph/0010570`), doi:10.1142/9789812811318_0006

Combes F., 2009, in Jogee S., Marinova I., Hao L., Blanc G. A., eds, Astronomical Society of the Pacific Conference Series Vol. 419, Galaxy Evolution: Emerging Insights and Future Challenges. p. 31 (arXiv:0901.0178)

Combes F., Sanders R. H., 1981, *A & A*, 96, 164

Combes F., Debbasch F., Friedli D., Pfenniger D., 1990a, *A & A*, 233, 82

Combes F., Debbasch F., Friedli D., Pfenniger D., 1990b, *A & A*, 233, 82

Conselice C. J., 2014, *Anu Rev. in Astronomy and Astrophysics.*, 52, 291

Conselice C. J., Wilkinson A., Duncan K., Mortlock A., 2016, *ApJ*, 830, 83

Costantin L., Corsini E. M., Méndez-Abreu J., Morelli L., Dalla Bontà E., Pizzella A., 2018, *MNRAS*, 481, 3623

Cox T. J., Loeb A., 2008, *MNRAS*, 386, 461

D'Onghia E., Burkert A., 2004, *ApJL*, 612, L13

D'Onghia E., Vogelsberger M., Faucher-Giguere C.-A., Hernquist L., 2010, *ApJ*, 725, 353

D'Onghia E., Vogelsberger M., Hernquist L., 2013, *ApJ*, 766, 34

D'Onghia E., Madau P., Vera-Ciro C., Quillen A., Hernquist L., 2016, *ApJ*, 823, 4

Dalcanton J. J., Spergel D. N., Summers F. J., 1997, *ApJ*, 482, 659

Das M., Sengupta C., Ramya S., Misra K., 2012, *MNRAS*, 423, 3274

Davies J. I., Phillipps S., Cawson M. G. M., Disney M. J., Kibblewhite E. J., 1988, *MNRAS*, 232, 239

De Lucia G., Springel V., White S. D. M., Croton D., Kauffmann G., 2006, *MNRAS*, 366, 499

De Rijcke S., Dejonghe H., Zeilinger W. W., Hau G. K. T., 2004, *A & A*, 426, 53

Deason A. J., Fattahi A., Frenk C. S., Grand R. J. J., Oman K. A., Garrison-Kimmel S., Simpson C. M., Navarro J. F., 2020, *MNRAS*, 496, 3929

Debattista V. P., 2014, *MNRAS*, 443, L1

Debattista V. P., Carollo C. M., Mayer L., Moore B., 2005, *ApJ*, 628, 678

Debattista V. P., Mayer L., Carollo C. M., Moore B., Wadsley J., Quinn T., 2006, *ApJ*, 645, 209

Dehnen W., 1993, *MNRAS*, 265, 250

Dehnen W., McLaughlin D. E., Sachania J., 2006, *MNRAS*, 369, 1688

Dekel A., Birnboim Y., 2006, *MNRAS*, 368, 2

Di Matteo P., Combes F., Melchior A. L., Semelin B., 2007, Astronomy and Astrophysics, 468, 61

Díaz-García S., Salo H., Laurikainen E., Herrera-Endoqui M., 2016, *A & A*, 587, A160

Díaz-García S., Salo H., Knapen J. H., Herrera-Endoqui M., 2019, *A & A*, 631, A94

Dobbs C., Baba J., 2014, *Publications of the Astronomical Society of Australia*, 31, e035

Dobbs C. L., Theis C., Pringle J. E., Bate M. R., 2010, *MNRAS*, 403, 625

Donner K. J., Thomasson M., 1994, *A & A*, 290, 785

Drory N., Fisher D. B., 2007a, *ApJ*, 664, 640

Drory N., Fisher D. B., 2007b, *ApJ*, 664, 640

Du W., McGaugh S. S., 2020, *AJ*, 160, 122

Dubinski J., 1998, *ApJ*, 502, 141

Dubinski J., Mihos J. C., Hernquist L., 1999, *ApJ*, 526, 607

Duc P.-A., Renaud F., 2013a, Tides in Colliding Galaxies. p. 327, doi:10.1007/978-3-642-32961-6_9

Duc P.-A., Renaud F., 2013b, in Souchay J., Mathis S., Tokieda T., eds, , Vol. 861, Lecture Notes in Physics, Berlin Springer Verlag. p. 327, doi:10.1007/978-3-642-32961-6_9

Duc P.-A., Cuillandre J.-C., Renaud F., 2018a, *MNRAS*, 475, L40

Duc P.-A., Cuillandre J.-C., Renaud F., 2018b, *MNRAS*, 475, L40

Dwek E., et al., 1995, *ApJ*, 445, 716

Eggen O. J., Lynden-Bell D., Sandage A. R., 1962, *ApJ*, 136, 748

Eliche-Moral M. C., González-García A. C., Balcells M., Aguerri J. A. L., Gallego J., Zamorano J., Prieto M., 2011, *A & A*, 533, A104

Erwin P., Debattista V. P., 2016, *ApJL*, 825, L30

Erwin P., Debattista V. P., 2017, *MNRAS*, 468, 2058

Erwin P., Sparke L. S., 2002, *AJ*, 124, 65

Erwin P., et al., 2015, *MNRAS*, 446, 4039

Erwin P., et al., 2021, *MNRAS*, 502, 2446

Faber S. M., Jackson R. E., 1976, *ApJ*, 204, 668

Fabricius M. H., Saglia R. P., Fisher D. B., Drory N., Bender R., Hopp U., 2012, *ApJ*, 754, 67

Fall S. M., Efstathiou G., 1980, *MNRAS*, 193, 189

Fathi K., Peletier R. F., 2003, *A & A*, 407, 61

Faure C., Siebert A., Famaey B., 2014, *MNRAS*, 440, 2564

Filippenko A. V., Ho L. C., 2003, *ApJL*, 588, L13

Fisher D. B., 2006a, in Kannappan S. J., Redfield S., Kessler-Silacci J. E., Landriau M., Drory N., eds, Astronomical Society of the Pacific Conference Series Vol. 352, New Horizons in Astronomy: Frank N. Bash Symposium ASP Conference Series, Vol. 352, Proceedings of the Conference Held 16-18 October, 2005 at The University of Texas, Austin, Texas, USA, p.237. p. 237

Fisher D. B., 2006b, in Kannappan S. J., Redfield S., Kessler-Silacci J. E., Landriau M., Drory N., eds, Astronomical Society of the Pacific Conference Series Vol. 352, New Horizons in Astronomy: Frank N. Bash Symposium. p. 237

Fisher D. B., 2006c, in Kannappan S. J., Redfield S., Kessler-Silacci J. E., Landriau M., Drory N., eds, Astronomical Society of the Pacific Conference Series Vol. 352, New Horizons in Astronomy: Frank N. Bash Symposium ASP Conference Series, Vol. 352, Proceedings of the Conference Held 16-18 October, 2005 at The University of Texas, Austin, Texas, USA, p.237. p. 237

Fisher D. B., 2006d, *ApJL*, 642, L17

Fisher D. B., 2006e, *ApJL*, 642, L17

Fisher D. B., Drory N., 2008a, *AJ*, 136, 773

Fisher D. B., Drory N., 2008b, *AJ*, 136, 773

Fisher D. B., Drory N., 2008c, *AJ*, 136, 773

Fisher D. B., Drory N., 2008d, in Funes J. G., Corsini E. M., eds, Astronomical Society of the Pacific Conference Series Vol. 396, Formation and Evolution of Galaxy Disks ASP Conference Series, Vol. 396, Proceedings of the conference held 1-5 October, 2007 at the Centro Convegni Matteo Ricci, Rome, Italy. Edited by José G. Funes, S.J., and Enrico Maria Corsini. San Francisco: Astronomical Society of the Pacific, 2008., p.309. p. 309

Fisher D. B., Drory N., 2008e, in Funes J. G., Corsini E. M., eds, Astronomical Society of the Pacific Conference Series Vol. 396, Formation and Evolution of Galaxy Disks ASP Conference Series, Vol. 396, Proceedings of the conference held 1-5 October, 2007 at the Centro Convegni Matteo Ricci, Rome, Italy. Edited by José G. Funes, S.J., and Enrico Maria Corsini. San Francisco: Astronomical Society of the Pacific, 2008., p.309. p. 309

Fisher D. B., Drory N., 2011a, *ApJL*, 733, L47

Fisher D. B., Drory N., 2011b, *ApJL*, 733, L47

Freeman K. C., 1970, *ApJ*, 160, 811

Frenk C. S., White S. D. M., Efstathiou G., Davis M., 1985, *Nature*, 317, 595

Freudenreich H. T., 1998, *ApJ*, 492, 495

Freudenreich H. T., et al., 1994, *ApJL*, 429, L69

Friedli D., Pfenniger D., 1990, in European Southern Observatory Conference and Workshop Proceedings. p. 265

Gadotti D. A., 2009a, *MNRAS*, 393, 1531

Gadotti D. A., 2009b, *MNRAS*, 393, 1531

Gadotti D. A., 2011, Memorie della Societa Astronomica Italiana Supplementi, 18, 69

Gadotti D. A., et al., 2020, *A & A*, 643, A14

Gaia Collaboration et al., 2018, *A & A*, 616, A11

Gajda G., Łokas E. L., Athanassoula E., 2018, *ApJ*, 868, 100

Gao H., Ho L. C., Barth A. J., Li Z.-Y., 2019, *ApJ Suppl.*, 244, 34

Gao H., Ho L. C., Barth A. J., Li Z.-Y., 2020, *ApJ Suppl.*, 247, 20

Gargiulo I. D., et al., 2019, *MNRAS*, 489, 5742

Genel S., et al., 2014, *MNRAS*, 445, 175

Genel S., et al., 2018, *MNRAS*, 474, 3976

Ghosh S., Jog C. J., 2015, *MNRAS*, 451, 1350

Ghosh S., Jog C. J., 2016, *MNRAS*, 459, 4057

Ghosh S., Jog C. J., 2022, *A & A*, 658, A171

Ghosh S., Saha K., Di Matteo P., Combes F., 2021, *MNRAS*, 502, 3085

Ghosh S., Debattista V. P., Khachaturyants T., 2022a, *MNRAS*, 511, 784

Ghosh S., Saha K., Jog C. J., Combes F., Matteo P. D., 2022b, *MNRAS*, 511, 5878

Gillessen S., et al., 2017, *ApJ*, 837, 30

Gómez F. A., Minchev I., O'Shea B. W., Beers T. C., Bullock J. S., Purcell C. W., 2013, *MNRAS*, 429, 159

Gott J. Richard I., Jurić M., Schlegel D., Hoyle F., Vogeley M., Tegmark M., Bahcall N., Brinkmann J., 2005, *ApJ*, 624, 463

Governato F., et al., 2009, *MNRAS*, 398, 312

Governato F., et al., 2010, *Nature*, 463, 203

Graham A. W., 2013, Elliptical and Disk Galaxy Structure and Modern Scaling Laws. pp 91–140, doi:10.1007/978-94-007-5609-0_2

Graham A. W., 2014, in Seigar M. S., Treuthardt P., eds, Astronomical Society of the Pacific Conference Series Vol. 480, Structure and Dynamics of Disk Galaxies. p. 185 (arXiv:1311.7207)

Graham A. W., Worley C. C., 2008, *MNRAS*, 388, 1708

Graham A. W., Erwin P., Caon N., Trujillo I., 2001, *ApJL*, 563, L11

Grand R. J. J., Kawata D., Cropper M., 2012, *MNRAS*, 421, 1529

Grossi M., Fernandes C. A. C., Sobral D., Afonso J., Telles E., Bizzocchi L., Paulino-Afonso A., Matute I., 2018, *MNRAS*, 475, 735

Guedes J., Mayer L., Carollo M., Madau P., 2013, *ApJ*, 772, 36

Gültekin K., et al., 2009, *ApJ*, 698, 198

Hammersley P. L., Garzón F., Mahoney T. J., López-Corredoira M., Torres M. A. P., 2000, *MNRAS*, 317, L45

Hart R. E., et al., 2016, *MNRAS*, 461, 3663

Häussler B., et al., 2007, *ApJ Suppl.*, 172, 615

Heller C. H., Shlosman I., 1994a, *ApJ*, 424, 84

Heller C. H., Shlosman I., 1994b, *ApJ*, 424, 84

Hernquist L., 1990, *ApJ*, 356, 359

Hetherington N. S., 1970, Leaflet of the Astronomical Society of the Pacific, 10, 313

Hockney R. W., Eastwood J. W., 1988, Computer simulation using particles

Hohl F., 1971, *ApJ*, 168, 343

Holmberg E., 1941, *ApJ*, 94, 385

Hopkins P. F., et al., 2009a, *MNRAS*, 397, 802

Hopkins P. F., et al., 2009b, *MNRAS*, 397, 802

Hopkins P. F., Cox T. J., Younger J. D., Hernquist L., 2009c, *ApJ*, 691, 1168

Hopkins P. F., et al., 2010a, *ApJ*, 715, 202

Hopkins P. F., et al., 2010b, *ApJ*, 715, 202

Horellou C., Koribalski B., 2007, *A & A*, 464, 155

Hubble E. P., 1925, Popular Astronomy, 33, 252

Hubble E. P., 1926, *ApJ*, 64, 321

Hubble E. P., 1936, Realm of the Nebulae

Hunter C., Toomre A., 1969, *ApJ*, 155, 747

Illingworth G., 1977, *ApJL*, 218, L43

Izquierdo-Villalba D., Bonoli S., Spinoso D., Rosas-Guevara Y., Henriques B. M. B., Hernández-Monteagudo C., 2019, *MNRAS*, 488, 609

Kaisin S. S., Karachentsev I. D., Hernandez-Toledo H., Gutierrez L., Karachentseva V. E., 2020, Astrophysical Bulletin, 75, 1

Karachentsev I. D., Karachentseva V. E., 2019, *MNRAS*, 485, 1477

Karachentsev I. D., Karachentseva V. E., Parnovskij S. L., 1993, Astronomische Nachrichten, 314, 97

Karachentsev I. D., Karachentseva V. E., Kudrya Y. N., 1999, Astronomy Letters, 25, 1

Karachentseva V. E., Karachentsev I. D., Kashibadze O. G., 2020, Astrophysics, 63, 151

Kataria S. K., Das M., 2018, *MNRAS*, 475, 1653

Kataria S. K., Das M., 2019, *ApJ*, 886, 43

Kataria S. K., Das M., Barway S., 2020, *A & A*, 640, A14

Kauffmann G., White S. D. M., Guiderdoni B., 1993a, *MNRAS*, 264, 201

Kauffmann G., White S. D. M., Guiderdoni B., 1993b, *MNRAS*, 264, 201

Kautsch S. J., Grebel E. K., Barazza F. D., Gallagher J. S. I., 2006, A catalog of edge-on disk galaxies. From galaxies with a bulge to superthin galaxies, Astronomy and Astrophysics, Volume 451, Issue 3, June I 2006, pp.1171-1171, doi:10.1051/0004-6361:20053981e

Kennicutt R. C. J., 1981, *AJ*, 86, 1847

Kereš D., Katz N., Weinberg D. H., Davé R., 2005, *MNRAS*, 363, 2

Keselman J. A., Nusser A., 2012, *MNRAS*, 424, 1232

Kim J. H., Peirani S., Kim S., Ann H. B., An S.-H., Yoon S.-J., 2014a, Astrophysical Journal, 789, 90

Kim J. H., Peirani S., Kim S., Ann H. B., An S.-H., Yoon S.-J., 2014b, *ApJ*, 789, 90

Kormendy J., 1977, *ApJ*, 218, 333

Kormendy J., 1982, *ApJ*, 257, 75

Kormendy J., 1983, *ApJ*, 275, 529

Kormendy J., 1993a, in Dejonghe H., Habing H. J., eds, IAU Symposium Vol. 153, Galactic Bulges. p. 209

Kormendy J., 1993b, in Dejonghe H., Habing H. J., eds, International Astronomical Union Symposium Vol. 153, Galactic Bulges. p. 209

Kormendy J., Fisher D. B., 2005, in Torres-Peimbert S., MacAlpine G., eds, Revista Mexicana de Astronomia y Astrofisica Conference Series Vol. 23, Revista Mexicana de Astronomia y Astrofisica Conference Series. pp 101–108 (arXiv:astro-ph/0507525)

Kormendy J., Kennicutt Robert C. J., 2004a, *Anu Rev. in Astronomy and Astrophysics.*, 42, 603

Kormendy J., Kennicutt Robert C. J., 2004b, *Anu Rev. in Astronomy and Astrophysics.*, 42, 603

Kormendy J., Kennicutt Robert C. J., 2004c, *Anu Rev. in Astronomy and Astrophysics.*, 42, 603

Kormendy J., Cornell M. E., Block D. L., Knapen J. H., Allard E. L., 2006a, *ApJ*, 642, 765

Kormendy J., Cornell M. E., Block D. L., Knapen J. H., Allard E. L., 2006b, *ApJ*, 642, 765

Kormendy J., Fisher D. B., Cornell M. E., Bender R., 2009, *ApJ Suppl.*, 182, 216

Kormendy J., Drory N., Bender R., Cornell M. E., 2010a, *ApJ*, 723, 54

Kormendy J., Drory N., Bender R., Cornell M. E., 2010b, *ApJ*, 723, 54

Krumholz M. R., Burkhart B., Forbes J. C., Crocker R. M., 2018, *MNRAS*, 477, 2716

Kumar A., Kataria S. K., 2022, *MNRAS*, 514, 2497

Kumar A., Das M., Kataria S. K., 2021, *MNRAS*, 506, 98

Kumar A., Das M., Kataria S. K., 2022a, *MNRAS*, 509, 1262

Kumar A., Das M., Kataria S. K., 2022b, *MNRAS*, 509, 1262

Kumar A., Ghosh S., Kataria S. K., Das M., Debattista V. P., 2022c, *MNRAS*, 516, 1114

Lackner C. N., Gunn J. E., 2012, *MNRAS*, 421, 2277

Lang M., Holley-Bockelmann K., Sinha M., 2014, *ApJL*, 790, L33

Laurikainen E., Salo H., 2001, *MNRAS*, 324, 685

Laurikainen E., Salo H., 2016a, in Laurikainen E., Peletier R., Gadotti D., eds, Astrophysics and Space Science Library Vol. 418, Galactic Bulges, Astrophysics and Space Science Library, Volume 418. ISBN 978-3-319-19377-9. Springer International Publishing Switzerland, 2016, p. 77. p. 77, doi:10.1007/978-3-319-19378-6_4

Laurikainen E., Salo H., 2016b, Observed Properties of Boxy/Peanut/Barlens Bulges. p. 77, doi:10.1007/978-3-319-19378-6_4

Laurikainen E., Salo H., Buta R., 2005, *MNRAS*, 362, 1319

Laurikainen E., Salo H., Buta R., Knapen J. H., 2007, *MNRAS*, 381, 401

Laurikainen E., Salo H., Buta R., Knapen J. H., 2009, *ApJL*, 692, L34

Laurikainen E., Peletier R., Gadotti D., eds, 2016, Galactic Bulges Astrophysics and Space Science Library Vol. 418, doi:10.1007/978-3-319-19378-6.

Lee J., et al., 2018, *ApJ*, 864, 69

Li C., Kauffmann G., Heckman T. M., White S. D. M., Jing Y. P., 2008, Monthly Notices of the Royal Astronomical Society, 385, 1915

Liao S., Chen J., Chu M. C., 2017, *ApJ*, 844, 86

Licquia T. C., Newman J. A., 2015, *ApJ*, 806, 96

Liszt H. S., Burton W. B., 1980, *ApJ*, 236, 779

Łokas E. L., 2018, *ApJ*, 857, 6

Łokas E. L., 2019a, *A & A*, 624, A37

Łokas E. L., 2019b, *A & A*, 629, A52

Łokas E. L., 2020, *A & A*, 634, A122

Łokas E. L., Athanassoula E., Debattista V. P., Valluri M., Pino A. d., Semczuk M., Gajda G., Kowalczyk K., 2014, *MNRAS*, 445, 1339

Łokas E. L., Ebrová I., del Pino A., Sybilska A., Athanassoula E., Semczuk M., Gajda G., Fouquet S., 2016, *ApJ*, 826, 227

Long S., Shlosman I., Heller C., 2014, *ApJL*, 783, L18

Lovell M. R., et al., 2018, *MNRAS*, 481, 1950

Lütticke R., Dettmar R. J., Pohlen M., 2000, *A & A Suppl.*, 145, 405

Majewski S. R., Skrutskie M. F., Weinberg M. D., Ostheimer J. C., 2003, *ApJ*, 599, 1082

Mapelli M., Moore B., Bland-Hawthorn J., 2008, *MNRAS*, 388, 697

Marinacci F., et al., 2018, *MNRAS*, 480, 5113

Martin G., Kaviraj S., Devriendt J. E. G., Dubois Y., Pichon C., 2018, *MNRAS*, 480, 2266

Martinez-Valpuesta I., Shlosman I., 2004, *ApJL*, 613, L29

Martinez-Valpuesta I., Shlosman I., Heller C., 2006a, *ApJ*, 637, 214

Martinez-Valpuesta I., Shlosman I., Heller C., 2006b, *ApJ*, 637, 214

Martinez-Valpuesta I., Aguerri J. A. L., González-García A. C., Dalla Vecchia C., Stringer M., 2017, *MNRAS*, 464, 1502

Masters K. L., et al., 2011, *MNRAS*, 411, 2026

Masters K. L., et al., 2019, *MNRAS*, 487, 1808

Mathur S. D., 1990, *MNRAS*, 243, 529

Matthews L. D., Gallagher John S. I., 1997, *AJ*, 114, 1899

Matthews L. D., van Driel W., 2000, *A & A Suppl.*, 143, 421

Mayer L., Wadsley J., 2004, *MNRAS*, 347, 277

McMillan P. J., 2011, *MNRAS*, 414, 2446

Melvin T., et al., 2014, *MNRAS*, 438, 2882

Méndez-Abreu J., Aguerri J. A. L., Corsini E. M., Simonneau E., 2008, *A & A*, 487, 555

Méndez-Abreu J., Debattista V. P., Corsini E. M., Aguerri J. A. L., 2014, *A & A*, 572, A25

Méndez-Abreu J., et al., 2017, *A & A*, 598, A32

Méndez-Abreu J., et al., 2018, *MNRAS*, 474, 1307

Mihos J. C., Hernquist L., 1994a, *ApJL*, 431, L9

Mihos J. C., Hernquist L., 1994b, *ApJL*, 431, L9

Mihos J. C., McGaugh S. S., de Blok W. J. G., 1997, *ApJL*, 477, L79

Mo H. J., Mao S., White S. D. M., 1998, *MNRAS*, 295, 319

Mo H., van den Bosch F. C., White S., 2010, Galaxy Formation and Evolution

Monaghan J. J., 1992, *Anu Rev. in Astronomy and Astrophysics.*, 30, 543

Monari G., Famaey B., Siebert A., 2015, *MNRAS*, 452, 747

Moreno J., et al., 2019, *MNRAS*, 485, 1320

Moster B. P., Macciò A. V., Somerville R. S., Johansson P. H., Naab T., 2010, *MNRAS*, 403, 1009

Moster B. P., Naab T., White S. D. M., 2013, *MNRAS*, 428, 3121

Muñoz-Mateos J. C., et al., 2015, *ApJ Suppl.*, 219, 3

Murali C., Katz N., Hernquist L., Weinberg D. H., Davé R., 2002, *ApJ*, 571, 1

Naab T., et al., 2014, *MNRAS*, 444, 3357

Naiman J. P., et al., 2018, *MNRAS*, 477, 1206

Navarro J. F., Frenk C. S., White S. D. M., 1996, *ApJ*, 462, 563

Nelson D., et al., 2015, Astronomy and Computing, 13, 12

Nelson D., et al., 2018, *MNRAS*, 475, 624

Nelson D., et al., 2019a, Computational Astrophysics and Cosmology, 6, 2

Nelson D., et al., 2019b, *MNRAS*, 490, 3234

Ness M., Lang D., 2016, *AJ*, 152, 14

Noguchi M., 1987, *MNRAS*, 228, 635

Norman C. A., Sellwood J. A., Hasan H., 1996, *ApJ*, 462, 114

O'Brien J. C., Freeman K. C., van der Kruit P. C., 2010, *A & A*, 515, A63

Ocvirk P., Pichon C., Teyssier R., 2008, *MNRAS*, 390, 1326

Oh S. H., Kim W.-T., Lee H. M., Kim J., 2008, *ApJ*, 683, 94

Oh S. H., Kim W.-T., Lee H. M., 2015a, *ApJ*, 807, 73

Oh S. H., Kim W.-T., Lee H. M., 2015b, *ApJ*, 807, 73

Okamoto T., 2013, *MNRAS*, 428, 718

Pardy S. A., D'Onghia E., Athanassoula E., Wilcots E. M., Sheth K., 2016, *ApJ*, 827, 149

Pedrosa S. E., Tissera P. B., 2015, *A & A*, 584, A43

Peebles P. J. E., 2020a, *MNRAS*, 498, 4386

Peebles P. J. E., 2020b, *MNRAS*, 498, 4386

Peng C. Y., 2003, GALFIT USER'S MANUAL. https://users.obs.carnegiescience.edu/peng/work/galfit/README.pdf

Peng C. Y., 2007, *ApJ*, 671, 1098

Peng C. Y., Ho L. C., Impey C. D., Rix H.-W., 2002a, *AJ*, 124, 266

Peng C. Y., Ho L. C., Impey C. D., Rix H.-W., 2002b, *AJ*, 124, 266

Peng C. Y., Ho L. C., Impey C. D., Rix H.-W., 2010a, *AJ*, 139, 2097

Peng C. Y., Ho L. C., Impey C. D., Rix H.-W., 2010b, *AJ*, 139, 2097

Peng C. Y., Ho L. C., Impey C. D., Rix H.-W., 2011, GALFIT: Detailed Structural Decomposition of Galaxy Images (ascl:1104.010)

Peters W., Kuzio de Naray R., 2019, *MNRAS*, 484, 850

Pettitt A. R., Wadsley J. W., 2018, *MNRAS*, 474, 5645

Pettitt A. R., Tasker E. J., Wadsley J. W., Keller B. W., Benincasa S. M., 2017, *MNRAS*, 468, 4189

Pfenniger D., Norman C., 1990, *ApJ*, 363, 391

Pillepich A., et al., 2018a, *MNRAS*, 473, 4077

Pillepich A., et al., 2018b, *MNRAS*, 475, 648

Pillepich A., et al., 2019, *MNRAS*, 490, 3196

Piontek F., Steinmetz M., 2011, *MNRAS*, 410, 2625

Planck Collaboration et al., 2016, *A & A*, 594, A13

Planck Collaboration et al., 2020, *A & A*, 641, A6

Poggio E., et al., 2018, *MNRAS*, 481, L21

Power C., Navarro J. F., Jenkins A., Frenk C. S., White S. D. M., Springel V., Stadel J., Quinn T., 2003, *MNRAS*, 338, 14

Puerari I., Block D. L., Elmegreen B. G., Frogel J. A., Eskridge P. B., 2000, *A & A*, 359, 932

Querejeta M., Eliche-Moral M. C., Tapia T., Borlaff A., Rodríguez-Pérez C., Zamorano J., Gallego J., 2015, *A & A*, 573, A78

Quillen A. C., Minchev I., Sharma S., Qin Y.-J., Di Matteo P., 2014, *MNRAS*, 437, 1284

Raha N., Sellwood J. A., James R. A., Kahn F. D., 1991a, *Nature*, 352, 411

Raha N., Sellwood J. A., James R. A., Kahn F. D., 1991b, *Nature*, 352, 411

Regan M. W., Teuben P. J., 2004, *ApJ*, 600, 595

Renaud F., Bournaud F., Duc P.-A., 2015, *MNRAS*, 446, 2038

Reshetnikov V., Combes F., 1996, *A & A Suppl.*, 116, 417

Reshetnikov V., Combes F., 1997a, *A & A*, 324, 80

Reshetnikov V., Combes F., 1997b, *A & A*, 324, 80

Robertson B., Yoshida N., Springel V., Hernquist L., 2004, *ApJ*, 606, 32

Rodriguez-Gomez V., et al., 2019, *MNRAS*, 483, 4140

Rubin V. C., Ford W. Kent J., 1970, *ApJ*, 159, 379

Saha K., 2015, *ApJL*, 806, L29

Saha K., Elmegreen B., 2016, *ApJL*, 826, L21

Saha K., Naab T., 2013, *MNRAS*, 434, 1287

Saha K., Martinez-Valpuesta I., Gerhard O., 2012a, *MNRAS*, 421, 333

Saha K., Martinez-Valpuesta I., Gerhard O., 2012b, *MNRAS*, 421, 333

Saha K., Gerhard O., Martinez-Valpuesta I., 2016a, *A & A*, 588, A42

Saha K., Gerhard O., Martinez-Valpuesta I., 2016b, *A & A*, 588, A42

Salo H., Laurikainen E., 2000, *MNRAS*, 319, 377

Salo H., et al., 2015, *ApJ Suppl.*, 219, 4

Samland M., Gerhard O. E., 2003, *A & A*, 399, 961

Sandage A., Visvanathan N., 1978, *ApJ*, 225, 742

Satyapal S., et al., 2016, *ApJ*, 827, 58

Savitzky A., Golay M. J. E., 1964, Analytical Chemistry, 36, 1627

Savorgnan G., Graham A. W., Marconi A., Sani E., Hunt L. K., Vika M., Driver S. P., 2013, *MNRAS*, 434, 387

Schönrich R., Dehnen W., 2018, *MNRAS*, 478, 3809

Scoville N. Z., Sanders D. B., 1987, H_2 in the Galaxy. p. 21, doi:10.1007/978-94-009-3861-8_2

Sellwood J. A., 1980, *A & A*, 89, 296

Sellwood J. A., Athanassoula E., 1986, *MNRAS*, 221, 195

Sellwood J. A., Carlberg R. G., 1984a, *ApJ*, 282, 61

Sellwood J. A., Carlberg R. G., 1984b, *ApJ*, 282, 61

Sellwood J. A., Gerhard O., 2020, *MNRAS*, 495, 3175

Sellwood J. A., Merritt D., 1994, *ApJ*, 425, 530

Semczuk M., Łokas E. L., del Pino A., 2017, *ApJ*, 834, 7

Semczuk M., Łokas E. L., D'Onghia E., Athanassoula E., Debattista V. P., Hernquist L., 2020, *MNRAS*, 498, 3535

Sersic J. L., 1968, Atlas de Galaxias Australes

Shapley H., Curtis H. D., 1921, Bulletin of the National Research Council, 2, 171

Shen J., Sellwood J. A., 2004, *ApJ*, 604, 614

Shen S., Mo H. J., White S. D. M., Blanton M. R., Kauffmann G., Voges W., Brinkmann J., Csabai I., 2003, *MNRAS*, 343, 978

Shen J., Rich R. M., Kormendy J., Howard C. D., De Propris R., Kunder A., 2010, *ApJL*, 720, L72

Sheth K., et al., 2008, *ApJ*, 675, 1141

Sheth K., et al., 2010, *Pub. Astron. Soc. Pac.*, 122, 1397

Sijacki D., Vogelsberger M., Genel S., Springel V., Torrey P., Snyder G. F., Nelson D., Hernquist L., 2015, *MNRAS*, 452, 575

Simard L., et al., 2002, *ApJ Suppl.*, 142, 1

Simard L., Mendel J. T., Patton D. R., Ellison S. L., McConnachie A. W., 2011, *ApJ Suppl.*, 196, 11

Sinha M., Holley-Bockelmann K., 2012a, *ApJ*, 751, 17

Sinha M., Holley-Bockelmann K., 2012b, *ApJ*, 751, 17

Sinha M., Holley-Bockelmann K., 2015a, arXiv e-prints, p. arXiv:1505.07910

Sinha M., Holley-Bockelmann K., 2015b, arXiv e-prints, p. arXiv:1505.07910

Smirnov A. A., Sotnikova N. Y., 2019, *MNRAS*, 485, 1900

Springel V., 2005a, User guide for GADGET-2. https://wwwmpa.mpa-garching.mpg.de/gadget/users-guide.pdf

Springel V., 2005b, *MNRAS*, 364, 1105

Springel V., 2010, *MNRAS*, 401, 791

Springel V., Yoshida N., White S. D. M., 2001a, *Nature Astronomy*, 6, 79

Springel V., White S. D. M., Tormen G., Kauffmann G., 2001b, *MNRAS*, 328, 726

Springel V., Frenk C. S., White S. D. M., 2006, *Nature*, 440, 1137

Springel V., et al., 2018, *MNRAS*, 475, 676

Stewart K. R., 2009, in Jogee S., Marinova I., Hao L., Blanc G. A., eds, Astronomical Society of the Pacific Conference Series Vol. 419, Galaxy Evolution: Emerging Insights and Future Challenges. p. 243

Stewart K. R., Bullock J. S., Wechsler R. H., Maller A. H., Zentner A. R., 2008, *ApJ*, 683, 597

Stilp A. M., Dalcanton J. J., Skillman E., Warren S. R., Ott J., Koribalski B., 2013, *ApJ*, 773, 88

Struck C., Dobbs C. L., Hwang J.-S., 2011, *MNRAS*, 414, 2498

Subramanian S., Ramya S., Das M., George K., Sivarani T., Prabhu T. P., 2016, *MNRAS*, 455, 3148

Sundelius B., Thomasson M., Valtonen M. J., Byrd G. G., 1987, *A & A*, 174, 67

Tabor M., Merrifield M., Aragón-Salamanca A., Cappellari M., Bamford S. P., Johnston E., 2017, *MNRAS*, 466, 2024

Tonini C., Mutch S. J., Croton D. J., Wyithe J. S. B., 2016, *MNRAS*, 459, 4109

Toomre A., 1964, *ApJ*, 139, 1217

Toomre A., Toomre J., 1972, *ApJ*, 178, 623

Torrey P., et al., 2019, *MNRAS*, 484, 5587

Vesperini E., Weinberg M. D., 2000, *ApJ*, 534, 598

Vogelsberger M., Genel S., Sijacki D., Torrey P., Springel V., Hernquist L., 2013, *MNRAS*, 436, 3031

Vogelsberger M., et al., 2014a, *MNRAS*, 444, 1518

Vogelsberger M., et al., 2014b, *Nature*, 509, 177

Walker I. R., Mihos J. C., Hernquist L., 1996, *ApJ*, 460, 121

Wang L., Kauffmann G., 2008, *MNRAS*, 391, 785

Wang W., Han J., Cautun M., Li Z., Ishigaki M. N., 2020a, Science China Physics, Mechanics, and Astronomy, 63, 109801

Wang J., Bose S., Frenk C. S., Gao L., Jenkins A., Springel V., White S. D. M., 2020b, *Nature*, 585, 39

Wegg C., Gerhard O., 2013, *MNRAS*, 435, 1874

Wegg C., Gerhard O., Portail M., 2015, *MNRAS*, 450, 4050

Weiland J. L., et al., 1994, *ApJL*, 425, L81

Weinberg M. D., 1991, *ApJ*, 373, 391

Weinberg M. D., 1992, *ApJ*, 384, 81

Weinberg M. D., Katz N., 2007, *MNRAS*, 375, 460

Weinberger R., et al., 2017, *MNRAS*, 465, 3291

Weinzirl T., Jogee S., Khochfar S., Burkert A., Kormendy J., 2009a, *ApJ*, 696, 411

Weinzirl T., Jogee S., Khochfar S., Burkert A., Kormendy J., 2009b, *ApJ*, 696, 411

White S. D. M., Rees M. J., 1978a, *MNRAS*, 183, 341

White S. D. M., Rees M. J., 1978b, *MNRAS*, 183, 341

Widrow L. M., Gardner S., Yanny B., Dodelson S., Chen H.-Y., 2012, *ApJL*, 750, L41

Widrow L. M., Barber J., Chequers M. H., Cheng E., 2014, *MNRAS*, 440, 1971

Willett K. W., et al., 2013, *MNRAS*, 435, 2835

Williams M. E. K., et al., 2013, *MNRAS*, 436, 101

Wu Y.-T., Pfenniger D., Taam R. E., 2016, *ApJ*, 830, 111

Xiang K. M., et al., 2021, *ApJ*, 909, 125

Xu Y., Newberg H. J., Carlin J. L., Liu C., Deng L., Li J., Schönrich R., Yanny B., 2015, *ApJ*, 801, 105

Yee H. K. C., Ellingson E., 1995, Astrophysical Journal, 445, 37

Young C. K., Currie M. J., 1998, *A & A Suppl.*, 127, 367

Yu X., Bian F., Krumholz M. R., Shi Y., Li S., Chen J., 2021, *MNRAS*, 505, 5075

Yurin D., Springel V., 2014, *MNRAS*, 444, 62

Zepf S. E., 1997, *Nature*, 390, 377

Zwicky F., 1933, Helvetica Physica Acta, 6, 110

de Grijs R., Peletier R. F., 1997, *A & A*, 320, L21

de Swart J. G., Bertone G., van Dongen J., 2017, Nature Astronomy, 1, 0059

de Vaucouleurs G., 1948, Annales d'Astrophysique, 11, 247

de Vaucouleurs G., 1959a, Handbuch der Physik, 53, 275

de Vaucouleurs G., 1959b, Handbuch der Physik, 53, 275

de Vaucouleurs G., 1959c, Handbuch der Physik, 53, 311

van Albada T. S., Bahcall J. N., Begeman K., Sancisi R., 1985, *ApJ*, 295, 305

van den Bergh S., 2007, *Nature*, 445, 265

van den Bergh S., Abraham R. G., Ellis R. S., Tanvir N. R., Santiago B. X., Glazebrook K. G., 1996, *AJ*, 112, 359

van den Bosch F. C., Burkert A., Swaters R. A., 2001, *MNRAS*, 326, 1205

van der Hulst J. M., Skillman E. D., Smith T. R., Bothun G. D., McGaugh S. S., de Blok W. J. G., 1993, *AJ*, 106, 548